

1. Let  $f(x) = \begin{cases} \frac{x+1}{(x-4)(x-2)^2}, & \text{if } x < 2 \\ 1/2 & \text{if } x = 2 \\ \frac{x^2-2x}{x^2-4}, & \text{if } x > 2 \end{cases}$

a) Determine whether  $f$  is left continuous at 2. Show your work with limits. Explain (means use words and the definition!)

$\lim_{x \rightarrow 2^-} f(x) \stackrel{x < 2}{=} \lim_{x \rightarrow 2^-} \frac{(x+1) \rightarrow 3}{(x-4)(x-2)^2} = -\infty$ . And  $f(2) = 1/2$ .  
 $\hookrightarrow -2(0)^2 = 0$

So  $\lim_{x \rightarrow 2^-} f(x) = -\infty \neq f(2)$ . Therefore not left cont.  
 not equal  $\uparrow$   
 $-\infty \neq 1/2$

b) Determine whether  $f$  is right continuous at 2. Show your work with limits and explain

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{x^2-2x}{x^2-4} \stackrel{\text{Factor}}{=} \lim_{x \rightarrow 2^+} \frac{x(x-2)}{(x-2)(x+2)} = \lim_{x \rightarrow 2^+} \frac{x}{x+2} = \frac{2}{4} = \frac{1}{2}$

Since  $\lim_{x \rightarrow 2^+} f(x) = f(2)$ ,  $f$  is right continuous  
 $1/2 = 1/2 \checkmark$

c) Determine whether  $f$  is continuous at 2. Explain using your work above.

No...  $f$  is not continuous at  $x=2$  b/c  $f$  is not both left and right continuous @ 2. OR since  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$ ,  $\lim_{x \rightarrow 2} f(x)$  DNE.  $\therefore f$  not cont @ 2.

d) Determine whether  $f$  has a removable discontinuity at 2. Explain with appropriate limits or limit language.

No RD.  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$ , so  $\lim_{x \rightarrow 2} f(x)$  DNE  
 $-\infty \neq 1/2$

$\therefore$  Cannot be a RD @  $x=2$

e) Determine whether  $f$  has a VA at 2. Explain with appropriate limits or limit language.

Yes, b/c  $\lim_{x \rightarrow 2^-} f(x) = -\infty$ ,  $f(x)$  has a VA @  $x=2$ .  
 Rt Cont.

f) Use interval notation to describe the intervals of continuity for  $f$ .  $(-\infty, 2) \cup [2, \infty)$

2. (Review your Quiz.) Determine  $\lim_{x \rightarrow -\infty} \frac{1-2x}{\sqrt{36x^2+x-1}}$ .  $\downarrow$  HP  $= \lim_{x \rightarrow -\infty} \frac{-2x}{\sqrt{36x^2}} = \lim_{x \rightarrow -\infty} \frac{-2x}{16|x|}$

$(x < 0)$   
 $= \lim_{x \rightarrow -\infty} \frac{-2x}{-6x} = \frac{1}{3}$

$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

3. Evaluate these limits. Indicate where you used the composition rule for limits and where you used continuity of trig or other functions. See Examples 10.1.21, 10.1.22, and 10.1.26 in the online Notes for Day 10.

a)  $\lim_{x \rightarrow 1} \sin(4x^3 - 2x^2 + x - 3)$   $\sin x$  is continuous and  $4x^3 - 2x^2 + x - 3$  is a continuous poly. So using composite continuity

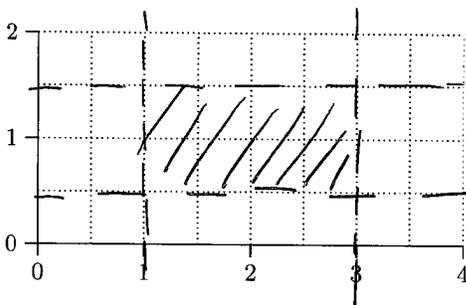
$\lim_{x \rightarrow 1} \sin(4x^3 - 2x^2 + x - 3) = \sin(\lim_{x \rightarrow 1} (4x^3 - 2x^2 + x - 3)) = \sin(0) = 0$

b)  $\lim_{x \rightarrow 0} \frac{e^x}{6x^2 + 1 + 4 \cos x}$ ,  $e^x$  is cont,  $6x^2 + 1$  is a poly (cont),  $4 \cos x$  is cont (trig)

So  $\lim_{x \rightarrow 0} \frac{e^x}{6x^2 + 1 + 4 \cos x} = \lim_{x \rightarrow 0} \frac{e^x}{6x^2 + 1 + 4 \cos x} = \frac{e^0}{6(0)^2 + 1 + 4 \cos(0)} = \frac{1}{1+4} = \frac{1}{5}$

4. a) Test Review: Graph the set of points that satisfies both  $|x-2| < 1$  and  $|y-1| < 0.5$ .

$|x-2| < 1$  so  
 $-1 < x-2 < 1$   
 $1 < x < 3$



$|y-1| < 1/2$   
 $-1/2 < y-1 < 1/2$   
 $1/2 < y < 3/2$

5. Use the Intermediate Value Theorem to show that  $f(x) = x^2 \ln x = 5$  at some point on the interval  $[1, e]$ . See Examples 10.1.28-30 in the online notes. Be sure to explain carefully.

Check the hypotheses:

• Is  $f$  continuous on  $[1, e]$ . Yes, it is a product of continuous functions,  $x^2$  (poly) and  $\ln x$  ... so the product is continuous

• Is  $L=5$  between  $f(1)$  and  $f(e)$ ?

$f(1) = 1^2 \ln 1 = 1 \cdot 0 = 0$

$f(e) = e^2 \ln e = e^2 \cdot 1 = 7.389$

} so 5 is between  $f(1)$  and  $f(e)$

Conclusion: by the IVT, there is some number  $c$  between 1 and  $e$  so that  $f(c) = 5$ .