Math 130 Day ~~14~~ 15, Hand In. Name: Answers

You need to do lots of differentiation practice! Do the practice on the other sheet. WeBWorK Set Day 15. Due Tuesday.

1. Use the derivative rules to determine these derivatives. Convert to exponent notation if needed. **Mathematical Grammar.** Write your answers in the form $\frac{d}{dx}[f(x)] = f'(x)$ (or other appropriate variable).

Model Problem: Find the derivative of $f(t) = \frac{1}{t^{4/3}}$.

Solution: First rewrite f in exponent form: $f(t) = \frac{1}{t^{4/3}} = t^{-4/3}$. So $\frac{d}{dt}[t^{-4/3}] \overset{\text{Power Rule}}{=} -\frac{4}{3}t^{-7/3}$. Note the use of $\frac{d}{dt}$ since the variable is t.

a) P. 151 #8

$$\frac{d}{dt}(t^{11}) = \boxed{11t^{10}}$$

b) P. 151 #10 (be careful) e^3 is a constant

$$\frac{d}{dt}(e^3) = \boxed{0}$$

c) P. 151 #12

$$\frac{d}{dv}(v^{100}) = \boxed{100v^{99}}$$

d) P. 151 #18

$$\frac{d}{ds}\left(\frac{\sqrt{s}}{4}\right) = \frac{d}{ds}\left(\frac{1}{4}s^{1/2}\right) = \frac{1}{4}\cdot\frac{1}{2}s^{-1/2} = \boxed{\frac{1}{8}s^{-1/2}}$$

e) P. 151 #20

$$\frac{d}{dx}(6x^5 - x) = 6\cdot 5x^4 - 1 = \boxed{30x^4 - 1}$$

f) P. 151 #22

$$\frac{d}{dt}(6\sqrt{t} - 4t^3 + 9) = \frac{d}{dt}(6t^{1/2} - 4t^3 + 9) = 6\cdot\frac{1}{2}t^{-1/2} - 4\cdot 3t^2 + 0 = \boxed{3t^{-1/2} - 12t^2}$$

g) $$\frac{d}{dt}\left[\frac{10}{t^2}\right] = \frac{d}{dt}(10t^{-2}) = 10(-2)t^{-3} = \boxed{-20t^{-3}}$$

h) $$\frac{d}{ds}\left[\frac{1}{\sqrt[5]{s^2}} + 6\right] = \frac{d}{ds}(s^{-2/5} + 6) = -\frac{2}{5}s^{-7/5} + 0 = \boxed{-\frac{2}{5}s^{-7/5}}$$

i) What is the equation of the tangent line in part (e) at $(1, f(1))$?

$$f(1) = 6(1)^5 - 1 = 5$$

$$\text{slope} = m = f'(1) = 30(1^4) - 1 = 29$$

Should be a number

$$y - 5 = 29(x - 1)$$

or $y = 29x - 29 + 5$ $\boxed{y = 29x - 24}$

2. Remember that the derivative represents an instantaneous rate of change. Suppose the position of an object moving along a line is given by $s(t) = 2t^2 - 12t + 10$, where position is measured in meters and time in seconds.

a) Determine the function that describes the *instantaneous velocity* of the object. (Use derivative rules.)

b) At what time(s) t is *instantaneous velocity* 0?

power, sum, const mult rules, constant

a) $s'(t) = \frac{d}{dt}\left[2t^2 - 12t + 10\right] = 2 \cdot 2t - 12 \cdot 1 + 0$

$= 4t - 12$

b) $s'(t) = 0 = 4t - 12$

$12 = 4t$

$3 = t$

$\boxed{t = 3}$

3. Rates of Growth. Remember that the derivative represents an instantaneous rate of change. The surface area (skin) s, measured in m^2, and the weight w, measured in kg, of a cow are related by the function $s = 0.9w^{2/3}$. What is the instantaneous rate of change in the surface area of the cow relative to the weight of the cow?

Inst Rate $= s'(w) = 0.9 \cdot \frac{2}{3} w^{-1/3} = 0.6\, w^{-1/3}$

4. a) Let $f(x) = (x^2 + 1)(x + 3)$. Determine $f'(x)$. Hint: First multiply $f(x)$, then use derivative properties.

$f(x) = x^3 + 3x^2 + x + 3$

power, const mult, sum rules
constant rule

So $f'(x) = \frac{d}{dx}\left[x^3 + 3x^2 + x + 3\right] = 3x^2 + 3 \cdot 2x + 1 + 0$

$= 3x^2 + 6x + 1$

5. Fill in this table using your knowledge of continuity an differentiability.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Continuous	Removable	Differentiable
2	1	1	1	1	yes	No	Yes
3	2	2	2	Not 2	No	Yes	No
4	1	1	1	2	No	Yes	No

6. Let $f(x) = \dfrac{1}{x+1}$. We do not have a derivative rule to handle this. Use the definition of the derivative as $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to determine $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{x+1-x-h-1}{(x+h+1)(x+1)h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}^{\,-1}}{(x+h+1)(x+1)\cancel{h}} = \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+1)(x+1)}$$

$$= \frac{-1}{(x+1)^2} \quad \text{or} \quad -(x+1)^{-2}$$

7. Let $f(x) = \sqrt{x+1}$. We do not have a derivative rule to handle this. Use the definition of the derivative as a limit (as above) to find the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$= \lim_{h\to 0} \frac{x+h+1-x-1}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} = \frac{1}{\sqrt{x+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$