

Math 130 Day 22

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00-6:00, 7:00-10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: <http://math.hws.edu/~mitchell/Math130F16/index.html>.

Practice

Monday

1. Practice Problems for Test 2 are now online. Answers posted by Wednesday morning.
2. Read/Re-read **Chapter 3.9** on Derivatives of Logs and Exponentials. Review the **online notes** (they are now back). We will finish this next time and also review Implicit Differentiation. (So review Chapter 3.8)
3. a) Page 199 #3, 9, 11, 15 (simplify first with a log rule), 17, 19(a classic).
b) Assuming we get this far: Page 199 #17 and 19.
4. More implicit differentiation problems. The answers are below. For each relation, find $\frac{dy}{dx}$.

a) $3y + \ln y = x^2 - x$ b) $\ln(x + y) = 2x$

c) Find the tangent line to the curve in (a) at the point (2, 1)

Answers

a) $\frac{dy}{dx} = \frac{2x-1}{3+\frac{1}{y}} = \frac{2xy-y}{3y+1}$ b) $\frac{dy}{dx} = 2x + 2y - 1$ c) $y = \frac{2}{7}x + \frac{3}{7}$

Hand In at Lab: Name: _____

1. Determine $\frac{dy}{dx}$ if $\sin(x^2y^2) = 2x - 1$.

$$\frac{d}{dx}(\sin(x^2y^2)) = \frac{d}{dx}(2x-1)$$

$$\cos(x^2y^2) \left[2xy^2 + 2x^2y \frac{dy}{dx} \right] = 2$$

$$2xy^2 \cos(x^2y^2) + 2x^2y \cos(x^2y^2) \frac{dy}{dx} = 2$$

$$2x^2y \cos(x^2y^2) \frac{dy}{dx} = 2 - 2xy^2 \cos(x^2y^2)$$

$$\frac{dy}{dx} = \frac{2 - 2xy^2 \cos(x^2y^2)}{2x^2y \cos(x^2y^2)} = \frac{1 - xy^2 \cos(x^2y^2)}{x^2y \cos(x^2y^2)}$$

2. Determine the derivative of $f(x) = \ln|x^2 \sin x|$. Hint: Simplify using a log law before differentiating.

$$f(x) = \ln|x^2| + \ln|\sin x|$$

$$f'(x) = \frac{1}{x^2} \cdot 2x + \frac{1}{\sin x} \cdot \cos x$$

$$= \frac{2}{x} + \frac{\cos x}{\sin x} = \frac{2}{x} + \cot x$$

OVER

3. Find the derivative of $g(x) = \ln \sqrt{x^4 + 9x^2 + 6}$. Hint: Simplify using a log law before differentiating.

$$g(x) = \ln(x^4 + 9x^2 + 6)^{1/2} = \frac{1}{2} \ln(\underbrace{x^4 + 9x^2 + 6}_u)$$

$$\begin{aligned} \text{So } g'(x) &= \frac{1}{2} \cdot \frac{1}{u} \cdot \frac{du}{dx} \\ &= \frac{1}{2} \cdot \frac{1}{x^4 + 9x^2 + 6} \cdot (4x^3 + 18x) = \frac{2x^3 + 9x}{x^4 + 9x^2 + 6} \end{aligned}$$

4. Find the derivative of $g(x) = 6x^5 \cdot 5^x$.

$$\begin{aligned} g'(x) &= 30x^4 \cdot 5^x + 6x^5 \cdot 5^x \ln 5 \\ &= 6x^4 \cdot 5^x (5 + x \ln 5) \end{aligned}$$

5. Find the derivative of $g(x) = 2^{\overbrace{\sin(3x^2)}^u} = 2^u$.

$$g'(x) = 2^u \ln 2 \frac{du}{dx} = 2^{\sin(3x^2)} \cdot \ln 2 \cdot \cos(3x^2) \cdot 6x$$

6. Use logarithmic differentiation to determine the derivative of $y = (x^2 + 1)^{4x^3}$.

$$\ln y = \ln(x^2 + 1)^{4x^3} = 4x^3 \ln(x^2 + 1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(4x^3 \ln(x^2 + 1))$$

$$\frac{1}{y} \frac{dy}{dx} = 12x^2 \ln(x^2 + 1) + 4x^3 \cdot \frac{1}{x^2 + 1} \cdot 2x$$

$$\frac{1}{y} \frac{dy}{dx} = 4x^2 \left(3 \ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right)$$

$$\begin{aligned} \frac{dy}{dx} &= y \cdot 4x^2 \left(3 \ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right) \\ &= (x^2 + 1)^{4x^3} \cdot 4x^2 \left(3 \ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right) \end{aligned}$$