Day 23 Hand In Next Class. Name: _____

0. WeBWorK Day 23 due **Tuesday**. Practice Problems for Exam 2 are now Online. Covers material through today's class.

1. Fill in the exact values (no decimals) in the table on the left. Then use your first table to fill in the selected values in the second table. Remember, inverses reverse the input and output values of the original functions.

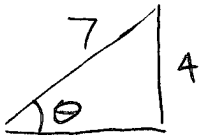
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

x	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\arcsin x$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\arccos x$	$\pi/2$	$\pi/3$	$\pi/4$	$\pi/6$	0

2. These are NOT derivative questions. (See class and online notes.) **Show your work using triangles** to help determine simpler expressions for

a) tan(arcsin 4/7) [WeBWorK Set Day 23, #4]

means $\sin\theta = 4/7 = \frac{opp}{hyp}$

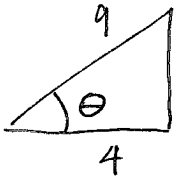


$\sqrt{7^2-4^2} = \sqrt{33}$

$\tan(\arcsin 4/7) = \tan\theta = \frac{4}{\sqrt{33}} = \frac{opp}{adj}$

b) sin(arccos(4/9)) [WeBWorK Set Day 23, #4]

$\cos\theta = 4/9 = \frac{adj}{hyp}$

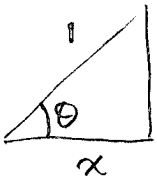


$\sqrt{9^2-4^2} = \sqrt{65}$

$\sin(\arccos(4/9)) = \sin\theta = \frac{\sqrt{65}}{9}$

c) sin(arccos(x)) [WeBWorK Set Day 23, #7]

$\cos\theta = x = \frac{adj}{hyp}$



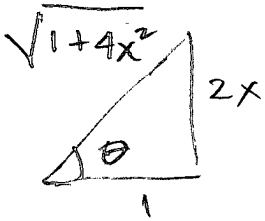
$\sqrt{1^2-x^2} = \sqrt{1-x^2}$

$\sin(\arccos(x)) = \sin\theta = \frac{\sqrt{1-x^2}}{1}$

$= \sqrt{1-x^2}$

d) cos(arctan($2x$)) [WeBWorK Set Day 23, #7]

$\tan\theta = 2x = \frac{opp}{adj}$



$\cos(\arctan(2x)) = \cos\theta = \frac{1}{\sqrt{1+4x^2}}$

3. Determine the derivatives of the following inverse trig functions. Use correct 'grammar.'

a) $y = \arcsin(4x^2)$ $= \arcsin u$, $u = 4x^2$, $\frac{du}{dx} = 8x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} = \frac{1}{\sqrt{1-16x^4}} \cdot 8x = \frac{8x}{\sqrt{1-16x^4}}$$

product rule

b) $y = \sin(2x)\arcsin(2x)$ [WeBWorK Set Day 23, #10]

$$\frac{dy}{dx} = \overbrace{\cos(2x)}^{f'} \cdot 2 \cdot \overbrace{\arcsin(2x)}^{g} + \sin(2x) \cdot \frac{1}{\sqrt{1-4x^2}} \cdot 2$$

$$= 2\left[\cos(2x)\arcsin(2x) + \frac{\sin(2x)}{\sqrt{1-4x^2}}\right]$$

c) $y = \arcsin(e^{7x})$ [Similar to WeBWorK Set Day 23, #8]

$y = \arcsin u$, $u = e^{7x}$, $\frac{du}{dx} = 7e^{7x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} = \frac{1}{\sqrt{1-e^{14x}}} \cdot 7e^{7x} = \frac{7e^{7x}}{\sqrt{1-e^{14x}}}$$

d) $y = \arctan(6x^2)$ $= \arctan u$, $u = 6x^2$, $\frac{du}{dx} = 12x$

$$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx} = \frac{1}{1+36x^4} \cdot 12x = \frac{12x}{1+36x^4}$$

e) $y = \arctan(\ln x)$ [WeBWorK Set Day 23, #15]

$y = \arctan u$, $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{1+u^2}\frac{du}{dx} = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x} = \frac{1}{x(1+\ln^2 x)}$$

4. Determine the derivatives of the following functions. Use correct 'grammar.'

a) $y = 8^{\arctan(x^2)}$

$$\frac{dy}{dx} = D_x(8^u) = 8^u \ln 8 \cdot \frac{du}{dx}$$

$$= 8^{\arctan(x^2)} \cdot \ln 8 \cdot \frac{1}{1+(x^2)^2} \cdot 2x$$

$$= \frac{8^{\arctan(x^2)} \cdot \ln 8 \cdot 2x}{1+x^4}$$

b) $y = (\arcsin x)^{\cos x}$

$$\ln y = \ln(\arcsin x)^{\cos x} = \cos x \ln(\arcsin x)$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \cdot \ln(\arcsin x) + \cos x \cdot \frac{1}{\arcsin x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = y\left[-\sin x \ln(\arcsin x) + \frac{\cos x}{\arcsin x \cdot \sqrt{1-x^2}}\right]$$

$$\frac{dy}{dx} = (\arcsin x)^{\cos x}\left[-\sin x \cdot \ln(\arcsin x) + \frac{\cos x}{\arcsin x \cdot \sqrt{1-x^2}}\right]$$