Math 130 Day 24, Hand In. Name: Answers

0. Work on WeBWorK set Day 24 due Thursday night. This reviews inverse trig and logarithmic differentiation and is good practice for the exam on this recent material. Now try this practice problem before working on the actual problems to hand in. Assume that x, y, θ, r, V, S, and h are functions of t. Find the derivatives with respect to t of each of these relations. (Check: Answers on the previous page.)

a) $V = x^2 y$ b) $S = 2\pi r^2 + 2\pi r h$ c) $\theta = \arctan \frac{x}{4}$

1. Determine the derivatives of the following functions.

a) (WeBWorK Set Day 24, #3.) $x^3 \arcsin(4x)$

product

$$D_x(x^3 \arcsin(4x)) = 3x^2 \arcsin(4x) + x^3 \cdot \frac{1}{\sqrt{1-16x^2}} \cdot 4$$

$$= x^2 \left(3\arcsin(4x) + \frac{4x}{\sqrt{1-16x^2}}\right)$$

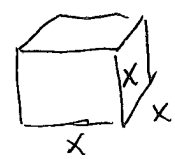
$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

b) (WeBWorK Set Day 24, #4.) $\arctan\left(\sqrt{4x^2-1}\right)$

$$\frac{dy}{dx} = \frac{1}{1+u^2}\frac{du}{dx} = \frac{1}{1+(\sqrt{4x^2-1})^2} \cdot \frac{1}{2}(4x^2-1)^{-1/2} \cdot 8x = \frac{4x}{1+4x^2-1} \cdot \frac{1}{\sqrt{4x^2-1}}$$

$$= \frac{4x}{4x^2\sqrt{4x^2-1}} = \frac{1}{x\sqrt{4x^2-1}}$$

2. Page 227 #10. Fill in the outline used in class.



1) Given (known) rate(s): $\frac{dx}{dt} = 2$ cm/s

2) Unknown rate: $\left.\frac{dV}{dt}\right|_{x=50}$

3) Relation: $V = x^3$

4) Rate-ify (take derivative):

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

5) Substitute known values to determine the unknown rate.

$$\left.\frac{dV}{dt}\right|_{x=50} = 3 \cdot (50)^2 \cdot 2 = 15{,}000 \text{ cm}^3/\text{s}$$

3. An animated rectangle in a Baby Einstein video is changing. Find the rate of change in its area A if $dh/dt = -2$ cm/s and $d\ell/dt = 3$ cm/s at the instant when $h = 12$ cm and $\ell = 5$ cm. Use the method in problem 2.

Given Rates: $\frac{dh}{dt} = -2\text{cm/s}$, $\frac{d\ell}{dt} = 3\text{cm/s}$

Unknown Rate: $\left.\frac{dA}{dt}\right|_{\substack{h=12\\ \ell=5}}$

Relation: $A = \ell h$

Rate-ify: $\frac{dA}{dt} = h\frac{d\ell}{dt} + \ell\frac{dh}{dt}$

Substitute: $\left.\frac{dA}{dt}\right|_{\substack{h=12\\ \ell=5}} = 12\cdot(3) + 5(-2) = 26\text{ cm}^2/\text{s}$

4. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Find the rate of change in the volume if $dr/dt = 2$ in/min and $dh/dt = 6$ in/min at the time when $r = 6$ in and $h = 18$ in. Use the method in problem 2.

Given Rates: $\frac{dr}{dt} = 2\text{in/m}$, $\frac{dh}{dt} = 6\text{in/min}$

Unknown Rate: $\left.\frac{dV}{dt}\right|_{\substack{r=6\\ h=18}}$

Relation: $V = \frac{1}{3}\pi r^2 h$

Rate-ify: $\frac{dV}{dt} = \frac{2}{3}\pi r h\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$

Substitute: $\left.\frac{dV}{dt}\right|_{\substack{r=6\\ h=18}} = \frac{2}{\cancel{3}}\pi(\cancel{6}^{2})(18)(2) + \frac{1}{\cancel{3}}\pi(6)^2\cdot\cancel{6}^{2}$

$= 144\pi + 72\pi$

$= 216\pi\text{ in}^3/\text{min}$