**Math 130. Hand In Monday: Name:** _Answers_

These problems will be assigned again on Friday.

0. Work on WeBWorK set Day 25. Seven Problems. Due Sunday night.

1. a) [WeBWorK #2] In an animated cartoon, a box with a square base is "growing" so that its height is increasing at 2 cm/s and its bottom edges are increasing at 3 cm/s. Determine how the volume of the box is changing when the height is 8 cm and the edge length is 5 cm. **Label each step as we did in class.**

Given Rates: $\frac{dh}{dt} = 2\,cm/s$ $\frac{dx}{dt} = 3\,cm/s$

Unknown Rate: $\frac{dV}{dt}\Big|_{\substack{h=8 \\ x=5}}$
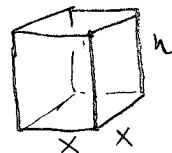
Relation: $V = x^2 h$

Rate-ify: $\frac{dV}{dt} = 2xh\frac{dx}{dt} + x^2\frac{dh}{dt}$

Subst: $\frac{dV}{dt}\Big|_{\substack{h=8 \\ x=5}} = 2(5)8(3) + 5^2(2) = 240 + 50 = 290\,cm^3/s$

b) How is the surface area of the box changing? **Label each step as we did in class.**

Find $\frac{dS}{dt}\Big|_{\substack{h=8 \\ x=5}}$ 4 rectangle sides

Relation: $S = 4xh + 2x^2$ ← square top & bottom

Rate-ify: $\frac{dS}{dt} = 4h\frac{dx}{dt} + 4x\frac{dh}{dt} + 4x\frac{dx}{dt}$

$\frac{dS}{dt}\Big|_{\substack{h=8 \\ x=5}} = 4(8)(3) + 4(5)(2) + 4(5)(3)$

$= 96 + 40 + 60$

$= 196\,cm^2/s$

2. a) [WeBWorK #4] The bottom of a ladder is sliding away from a wall at 2 ft/s. The ladder is 13 ft tall. How fast is the top of the ladder moving down the wall when the bottom of the ladder is 5 feet away from the wall. **Label each step as we did in class.**

Given Rate: $\dfrac{dx}{dt} = 2 \text{ ft/s}$

Find $\dfrac{dy}{dt}\Big|_{x=5}$

Relation: $x^2 + y^2 = 13^2$
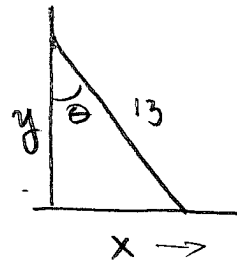
Rate $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \implies y\dfrac{dy}{dt} = -x\dfrac{dx}{dt}$

$$\dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt}$$

Subst:
when $x=5$, $y^2 = 13^2 - 5^2 = 169 - 25 = 144$, $y = 12$

$$\dfrac{dy}{dt}\Big|_{x=5} = -\dfrac{5}{12}\cdot 2 = -\dfrac{5}{6}\ \text{ft/s}$$

b) If θ is the angle between the ladder and the wall, how is θ changing at the same moment? **Label each step as we did in class.**

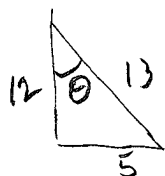Now find $\dfrac{d\theta}{dt}\Big|_{x=5}$

Relation: $\sin\theta = \dfrac{x}{13}$

Rate: $\cos\theta\dfrac{d\theta}{dt} = \dfrac{1}{13}\dfrac{dx}{dt}$

$$\dfrac{d\theta}{dt} = \dfrac{1}{13}\cdot\dfrac{1}{\cos\theta}\cdot\dfrac{dx}{dt} = \dfrac{\sec\theta}{13}\cdot\dfrac{dx}{dt}$$
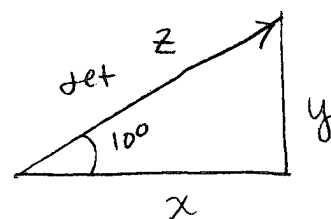
Subst:
when $x=5$ as above $y=12$; $\sec\theta = \dfrac{hyp}{opp} = \dfrac{13}{12}$

So $\dfrac{d\theta}{dt}\Big|_{x=5} = \dfrac{\sec\theta}{13}\cdot\dfrac{dx}{dt} = \dfrac{13}{12}\cdot\dfrac{1}{13}\cdot 2 = \dfrac{1}{6}\ \text{rad}$

3. Page 228 #20. Similar to Classwork Example 5 on the other side (also done in the online notes—look at them for guidance). Draw the triangle. Just do the first part: Determine how the height (altitude) of the triangle is changing.

Given $\dfrac{dz}{dt} = 550$ mi/hr

Find $\dfrac{dy}{dt}$

Relation: $\sin 10° = \dfrac{y}{z}$; $y = \sin 10° \cdot z$

Rate-ify: $\dfrac{dy}{dt} = \sin 10° \cdot \dfrac{dz}{dt} = \sin 10° \cdot (550) = 95.5$ mi/hr

constant (label pointing to $\sin 10°$)

Jet z, $10°$, y, x (triangle diagram)

Constant (label pointing to $\sin 10°$)

4. Page 228 #26. Same ideas as in Classwork Example 1 (also done in online notes). Draw the triangle. You know how the legs are changing, find how the hypotenuse is changing, in general. Then find how it is changing at 1 second (you can determine the lengths of the legs at this time).

Given $\dfrac{dx}{dt} = -18$ ft/s , $\dfrac{dy}{dt} = 20$ ft/s

gets shorter

y, x, z (triangle diagram)

Find $\dfrac{dz}{dt}\Big|_{t=1s}$
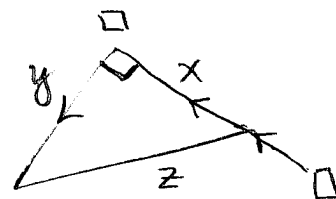
Relation: $z^2 = x^2 + y^2$

Rate-ify: $2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$

$\dfrac{dz}{dt} = \dfrac{x\dfrac{dx}{dt} + y\dfrac{dy}{dt}}{z}$

Rate × time

Subst: when $t=1$, $y = 20 \times 1 = 20$ ft ; $x = 90 - (18 \times 1) = 72$

$z = \sqrt{x^2 + y^2} = \sqrt{72^2 + 20^2}$

So $\dfrac{dz}{dt}\Big|_{t=1} = \dfrac{72(-18) + 20(20)}{\sqrt{72^2 + 20^2}} \approx -11.99$ ft/s

5. Page 229 #34. Use the same ideas as the Classwork Example 3 kite problem (also in online notes).

Given Rate: $\dfrac{dh}{dt} = 20\,ft/s$

Find $\dfrac{d\theta}{dt}\Big|_{h=400}$

Relation: $\tan\theta = \dfrac{h}{300}$

Rate: $\sec^2\theta\,\dfrac{d\theta}{dt} = \dfrac{1}{300}\dfrac{dh}{dt}$

when $h = 400$, $z = \sqrt{300^2 + 400^2} = 500$

$\sec^2\theta = \left(\dfrac{500}{300}\right)^2 = \dfrac{25}{9}$

Subst:

$\dfrac{25}{9}\dfrac{d\theta}{dt}\Big|_{h=400} = \dfrac{1}{300}\cdot 20$

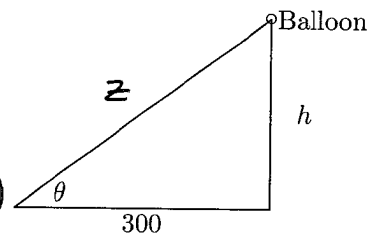$\dfrac{d\theta}{dt}\Big|_{h=400} = \dfrac{9}{25}\cdot\dfrac{2}{30} = \dfrac{3}{125} = .024\ rad/s$

Method 2:

$\theta = \arctan\left(\dfrac{h}{300}\right)$

Rate: $\dfrac{d\theta}{dt} = \dfrac{1}{1+\left(\frac{h}{300}\right)^2}\cdot\dfrac{1}{300}\cdot\dfrac{dh}{dt}$

Subst:

$\dfrac{d\theta}{dt}\Big|_{h=400} = \dfrac{1}{1+\left(\frac{400}{300}\right)^2}\cdot\dfrac{1}{300}\cdot 20$

$= .024\ rad/s$

Balloon diagram: triangle with hypotenuse z, vertical side h, horizontal side 300, angle θ.

6. a) [WeBWorK #7] Gravel is being dumped from a conveyor belt at a rate of 50 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter (watch out: what about the radius?) and height are always the same. How fast is the height of the pile increasing when the pile is 20 feet high? Recall that the volume of a right circular cone with height h and radius of the base r is given by $V = \frac{\pi}{3}r^2 h$. Hint: Write the volume in terms of the height only by using the relation between the height and the radius.

Given $\dfrac{dV}{dt} = 50\ ft^3/min$

Find $\dfrac{dh}{dt}\Big|_{h=20}$

Relation: $V = \dfrac{\pi}{3}r^2 h$, diameter $= h$, so $r = \dfrac{h}{2}$

So $V = \dfrac{\pi}{3}\left(\dfrac{h}{2}\right)^2 h = \dfrac{\pi}{3}\dfrac{h^3}{4} = \dfrac{\pi}{12}h^3$

Rate: $\dfrac{dV}{dt} = \dfrac{\pi}{4}h^2\dfrac{dh}{dt}$

Subst:

$50 = \dfrac{\pi}{4}(20)^2\cdot\dfrac{dh}{dt}\Big|_{h=20}$

$\dfrac{dh}{dt}\Big|_{h=20} = 50\cdot\dfrac{4}{\pi}\cdot\dfrac{1}{(20)^2} = \dfrac{1}{2\pi}\ ft/min$

b) XC: How is the lateral surface area of the cone changing at the same moment? (See inside cover of text for formula.) (Staple a new page.)