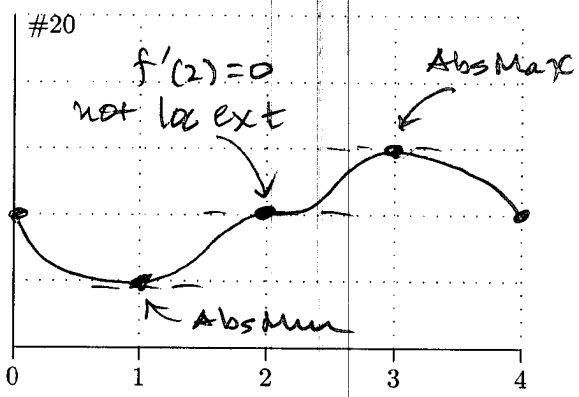
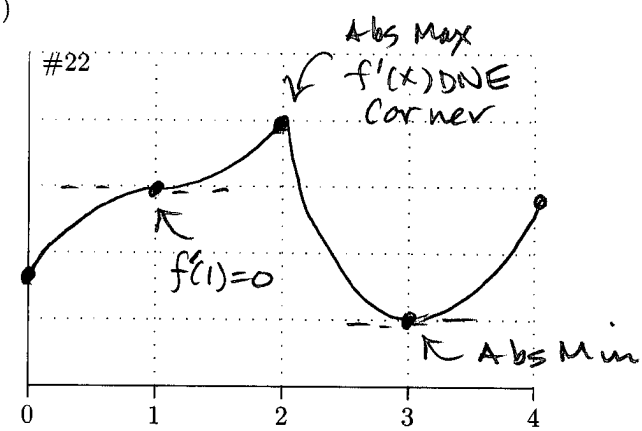
Math 130, Day 27. Hand In Wednesday: Name: Answers

0. There are two WeBWorK sets Day 26 which reviews critical numbers (due Wednesday) and Day 27 which reviews extrema (due Thursday). Both will help you with this homework assignment. Try some first.

1. Page 242–2433 #20 and 22. (Read the instructions on page 242.)





2. a) (Basics.) Find the critical number(s) of the function $f(x) = -2x^2 + 8x$. [WeBWorK Day 27, #1.]

Always defined

$f'(x) = -4x + 8 = 0$

$4x = 8$

$\boxed{x = 2}$

b) Use this information and the CIT to determine the extreme values (absolute max and absolute min) of $f(x)$ on the interval $[-1, 3]$.

Evaluate at CP: $f(2) = -8 + 16 = 8$ ← Abs Max @ $x = 2$

Evaluate at Endpts: $f(-1) = -2 - 8 = -10$ ← Abs Min @ $x = -1$

$f(3) = -18 + 24 = 6$

c) Determine the extreme values $f(x)$ on the interval ~~[0, 3]~~ [-3, 0].

Evaluate @ CP: None in interval

Evaluate @ Endpts: $f(-3) = -18 - 24 = -42$ ← Abs min @ $x = -3$

$f(0) = 0$ ← Abs Max @ $x = 0$

3. Use the same process as in the previous problem to determine the extreme values of $f(r) = \dfrac{3r}{r^2+1}$ on the interval $[-4, 0]$. [WeBWorK Day 27, #7.]

1) Find CPs: $f'(x) = \dfrac{3(r^2+1) - 3r(2r)}{(r^2+1)^2} = \dfrac{3-3r^2}{(r^2+1)^2} = \dfrac{3(1-r^2)}{(r^2+1)^2} = 0$

never 0

$r = \cancel{1}, -1$. Only $r = -1$ is in interval

2) Check Values

@ CP: $f(-1) = -\frac{3}{2}$ ← Abs Min @ $x = -1$

@ Endpts: $f(-4) = \dfrac{-12}{17}$

$f(0) = 0$ ← Abs Max @ $x = 0$

product rule

4. Page 243 #44. (Use the same process.) [WeBWorK Day 27, #5.] $f(x) = xe^{1-x/2}$ on $[0, 5]$

1) Find CP's: $f'(x) = e^{1-x/2} + x(-\frac{1}{2}e^{1-x/2})$

$= e^{1-x/2}(1 - \frac{1}{2}x) = 0 \Rightarrow 1 - \frac{1}{2}x = 0$

never 0

$\frac{1}{2}x = 1$

$\boxed{x = 2}$

2) Evaluate

@ CP: $f(2) = 2e^0 = 2$ ← Abs. Max @ $x = 2$

@ Endpts: $f(0) = 0$ ← Abs min @ $x = 0$

$f(5) = 5e^{1-5/2} = 5e^{-3/2} \approx 1.116$

7. [WeBWorK Day 27, #8.] If $y = x^{2x}$ on $[0.1, 1]$, find the absolute extreme values and the points at which they occur. Be careful. You will need a calculator to compare the values.

1) Use log. differentiation to find CP:

$$\ln y = \ln(x^{2x}) = 2x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + 2x \cdot \frac{1}{x} = 2\ln x + 2$$

Never 0

$$\frac{dy}{dx} = y(2\ln x + 2) = x^{2x}(2\ln x + 2) = 0$$

So $2\ln x + 2 = 0$

$2\ln x = -2$

$\ln x = -1$

CP: $\longrightarrow x = e^{-1} \approx 0.368$

in interval

2) Evaluate:

@ CP: $f(e^{-1}) = (e^{-1})^{2e^{-1}} \approx .479 \leftarrow$ Abs Min @ $x = e^{-1}$

@ Endpts: $f(0.1) = (.1)^{.2} = 0.631$

$f(1) = 1^2 = 1 \leftarrow$ Abs Max @ $x = 1$

Property	List all points in numerical Order
absolute max	4
relative max	2, 4, 7
relative max but not critical #	Impossible by CPT
not differentiable but relative max	2, 7
absolute min	0
absolute min but not relative min	0
critical number	2,3,4,5,7
critical # but not a relative extrema	5
critical # and $f'(x)$ DNE	2,3,5,7
$f'(x) = 0$ but not critical	Impossible by Definition of CP

5. Use the same process to determine the extreme values of $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$.

1) Find CPs: $f'(x) = 4x^3 - 4x = 4x(x^2-1) = 4x(x-1)(x+1) = 0$

$x = -1, 0, 1$

2) Evaluate:

@ CPs: $f(-1) = 1 - 2 = -1$

$f(0) = 0$

$f(1) = 1 - 2 = -1$

Abs Min @ $x = -1$ and 1

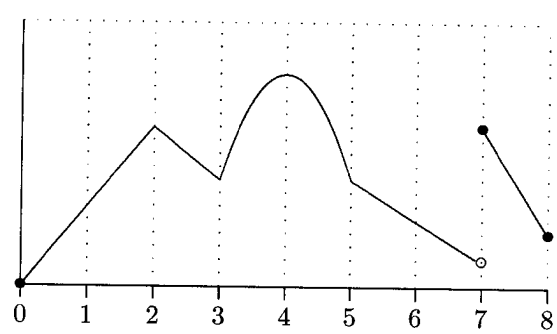
$f(-2) = 16 - 8 = 8$

$f(2) = 16 - 8 = 8$

Abs Max @ $x = -2$ and 2

6. f is defined on $[0, 8]$ as in the graph below. List x coordinates of all the points which satisfy the given property.

Property	List solutions in order
absolute max	
relative max	
relative max but not critical #	
not differentiable but relative max	
absolute min	
absolute min but not relative min	
critical number	
critical # but not a relative extrema	
critical # and $f'(x)$ DNE	
$f'(x) = 0$ but not critical	



The answers (in mirror writing) are given on the bottom of the next page. Grade yourself. Give yourself 1 point for each question you got entirely correct.

Score: /10