Math 130: Hand in at Lab 10. Name: Answers

-2. Definition. Let f be a function defined on an interval I containing the point c.

a) f has an **absolute (global) maximum** at c if $f(c) \geq f(x)$ for all x in D. The number $f(c)$ is the **maximum value** of f.

b) f has an **absolute (global) minimum** at c if $f(c) \leq f(x)$ for all x in D. The number $f(c)$ is the **minimum value** of f.

c) If $f(c) \geq f(x)$ for all x in some open interval containing c, then $f(c)$ is a **relative (local) maximum** value of f. (Or: f has a local max at c.)

d) If $f(c) \leq f(x)$ for all x in some open interval containing c, then f is a **relative (local) minimum** value of f. (Or: f has a local min at c.)

-1. MVT: The Mean Value Theorem. Assume that

1. f is continuous on the closed interval $[a, b]$;
2. f is differentiable on the open interval (a, b);

Then there is some point c in (a, b) so that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

0. EVT: Extreme Value Theorem. Let f be a continuous function on a closed interval $[a, b]$. Then f has both an absolute maximum value and an absolute minimum value on the interval $[a, b]$.

1. Designer Functions. Draw a function that satisfies the given conditions or **explain why this is impossible**. Make sure that your function is defined (has an output value) for every x in the given interval. [Each part is a separate problem.]

a) A continuous function on $[0, 10]$ which has an absolute min at $x = 2$ and has relative but not absolute max at $x = 6$.

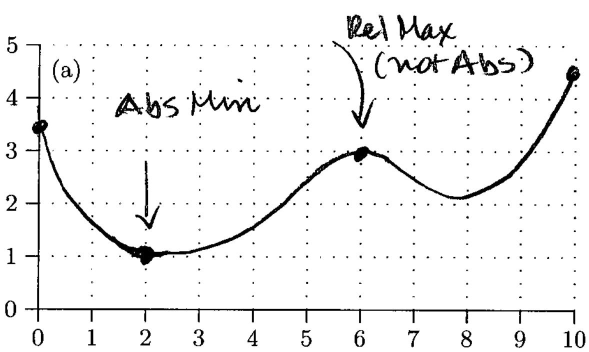
b) A function on $[0, 6]$ which has no absolute max.

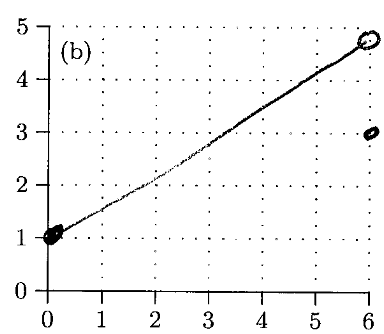
c) A continuous function on $(0, 6)$ which has no absolute max.

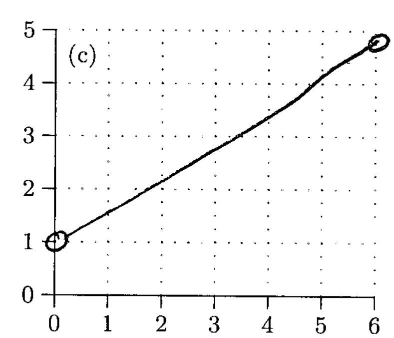
d) A function on $[0, 6]$ for which $f(0) = 3$ and $f(6) = -2$ and which is never 0.

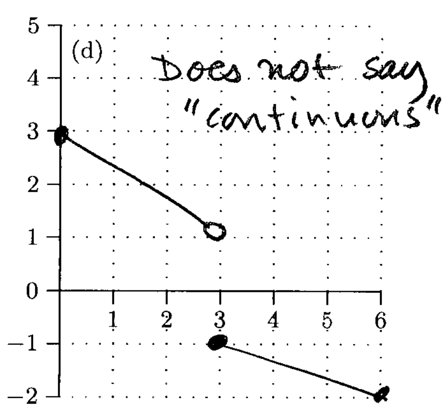
e) A differentiable function on $[0, 6]$ which has no absolute max. (Think: If f is differentiable, what else can you say about it?)

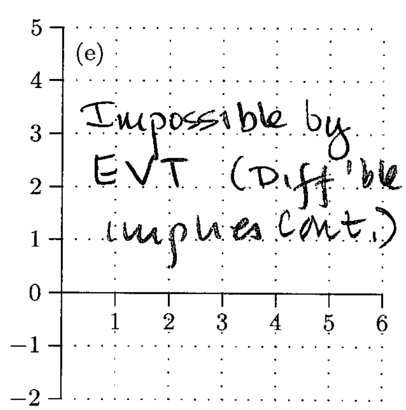
f) A function which illustrates the Mean Value Theorem. (Mark the tangent and secant lines.)

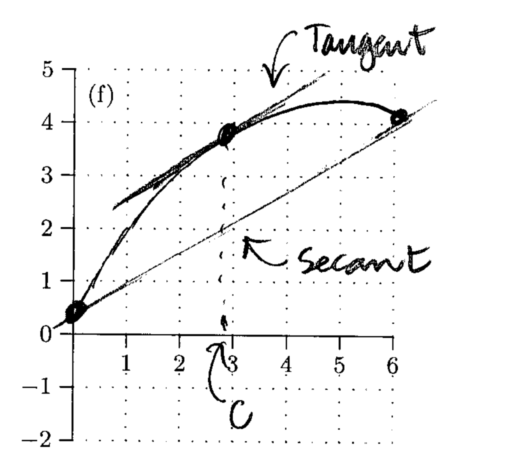








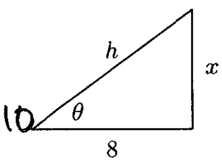




2. Suppose that one leg of a right triangle has a fixed length of 8 cm. Let x denote the other leg of the triangle. Assume that $dx/dt = 2\text{cm/sec}$. See figure below.

a) If h represents the length of the hypotenuse, find dh/dt when $x = 6$ cm.

Given $\frac{dx}{dt} = 2$ Find $\left.\frac{dh}{dt}\right|_{x=6}$ Relation $h^2 = x^2 + 8^2$



Rate: $2h\frac{dh}{dt} = 2x\frac{dx}{dt}$. When $x = 6$ $h = \sqrt{8^2+6^2} = 10$

$$\left.\frac{dh}{dt}\right|_{x=6} = \frac{x}{h}\frac{dx}{dt} = \frac{6}{10}(2) = 1.2 \text{ cm/s}$$

b) If θ is the angle shown, find $d\theta/dt$ when $x = 6$ cm.

Find $\left.\frac{d\theta}{dt}\right|_{x=6}$ Relation: $\tan\theta = \frac{x}{8}$

$$\theta = \arctan\frac{x}{8}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{8}\right)^2}\cdot\frac{1}{8}\cdot\frac{dx}{dt}$$

$$\left.\frac{d\theta}{dt}\right|_{x=6} = \frac{1}{1+\left(\frac{6}{8}\right)^2}\cdot\frac{1}{8}\cdot 2 = 16/100 = 4/25$$

rad/s

c) If A represents the area, find dA/dt when $x = 6$ cm.

Find $\left.\frac{dA}{dt}\right|_{x=6}$. Relation $A = \frac{1}{2}\cdot 8\cdot x = 4x$

Rate $\frac{dA}{dt} = 4\frac{dx}{dt}$

$$\left.\frac{dA}{dt}\right|_{x=6} = 4(2) = 8 \text{ cm}^2/\text{s}$$

3. a) Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$. Determine the absolute extreme points on $[-1, 2]$. WeBWorK Day28 #1.

1) Determine CP's, $f'(x) = x^3 + x^2 - 2x = x(x^2+x-2) = x(x-1)(x+2) = 0$

CP's: $x = 0, 1, \cancel{-2}$ not in interval

Evaluate f at CP's: $f(0) = 0$

$f(1) = \frac{1}{4} + \frac{1}{3} - 1 = -5/12$

Evaluate f at Endpts: $f(-1) = \frac{1}{4} - \frac{1}{3} - 1 = -13/12 \leftarrow$ Abs Min

$f(2) = 4 + 8/3 - 4 = 8/3 \leftarrow$ Abs Max

b) State the name of the theorem that you used: Closed Interval Thm