Math 130, Day 29: Answers

1. a) c is a critical point of f if c if f is defined at c and $f'(c) = 0$ or $f'(c)$ DNE.

b) $c = 3$ NOT a critical number of $f(x) = \frac{x}{x-3}$ because 3 is not in the domain of f.

2. a) Critical points: $f'(x) = 3x^2e^x + x^3e^x = e^x(x^3 + 3x^2) = e^x x^2(x+3) = 0$. So $x = 0,\ -3$

$f(-4) = (+)(+)(-) = (-)$ $\qquad$ $f(-2) = (+)(+)(+) = (+)$ $\qquad$ $f(1) = (+)(+)(+) = (-)$

		R Min		Neither	
	$--$	0	$+++$	0	$+++$
f'	Decr	-3	Incr	0	Incr

b) Critical points: $f'(x) = 5(x^2-1)^{2/3}2x = 10x[(x-1)(x+1)]^{2/3} = 0$. So $x = -1,\ 0,\ 1$

$f(-2) = (-)(+) = (-)$ $\qquad$ $f(-0.5) = (-)(+) = (-)$ $\qquad$ $f(0.5) = (+)(+) = (+)$ $\qquad$ $f(2) = (+)(+) = (+)$

		Neither		R Min		Neither	
	$---$	0	$---$	0	$+++$	0	$+++$
f'	Decr	-1	Decr	0	Incr	1	Incr

3. Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$.

a) Critical points: $f'(x) = x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x-1)(x+2) = 0$ at $x = 0,\ 1,\ -2$.

$f(-4) = (-)(-)(-) = (-)$ $\qquad$ $f(-1) = (-)(-)(+) = (+)$ $\qquad$ $f(0.5) = (+)(-)(+) = (-)$ $\qquad$ $f(2) = (+)(+)(+) = (+)$

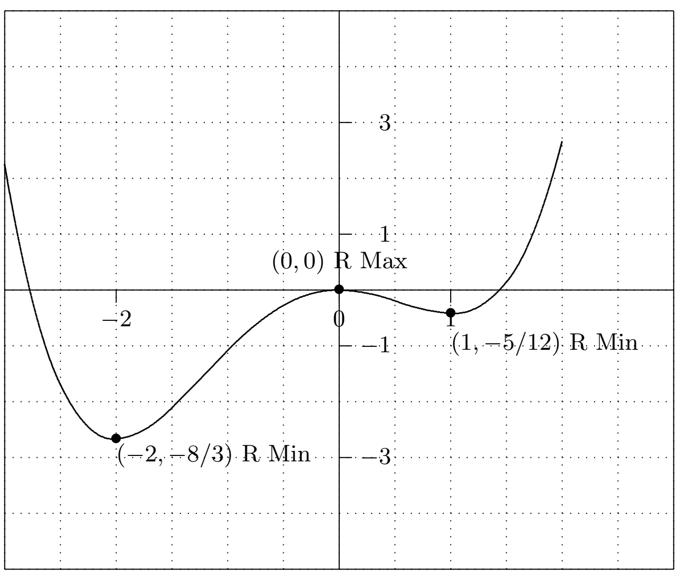
		R Min		R Max		R Min	
	$--$	0	$+++$	0	$-$	0	$+++$
h'	Decr	-2	Incr	0	Decr	1	Incr

To do this which theorem did you use? Increasing/Decreasing Test

b) Determine which critical points are local maxima, minima, and which are not extreme and mark this on your number line. (See above.)

To do this which theorem did you use? First Derivative Test

c)



At the critical numbers:
$f(0) = 0$,
$f(1) = -\frac{5}{12}$, and
$f(-2) = -\frac{8}{3}$.
Now plot these points and use the increasing/decreasing information from above.

d) Now using the information in parts (a), (b), and (c). make a sketch of the graph of f *without plotting any more individual points*. The shape of the graph should come from the information in parts (a), (b), and (c).

e) From part (a), the critical points of $f(x)$ in $[-1, 2]$ are $x = 0,\ 1$.

- Evaluate $f(x)$ at the critical points: $f(0) = 0$
- and $f(1) = -\frac{5}{12}$
- At the endpoints: $f(-1) = -\frac{13}{12}$
- and $f(2) = \frac{8}{3}$.
- Abs max is $\frac{8}{3}$ at $x = 2$; Abs min is $-\frac{13}{12}$ at $x = -1$.

Which theorem did you use? Closed Interval Theorem

4. a) $f(x) = \dfrac{2x+4}{x^2+5}$. Critical points:

$$f'(x) = \frac{2(x^2+5)-(2x+4)(2x)}{(x^2+5)^2} = \frac{-2x^2-8x+10}{(x^2+5)^2} = \frac{-2(x^2+4x-5)}{(x^2+5)^2} = \frac{-2(x+5)(x-1)}{(x^2+5)^2} = 0 \qquad \text{at } x = -5,\ 1.$$

$$f(-6) = (-)(-)(-) = (-) \qquad f(0) = (-)(-)(+) = (+) \qquad f(2) = (-)(+)(+) = (-)$$

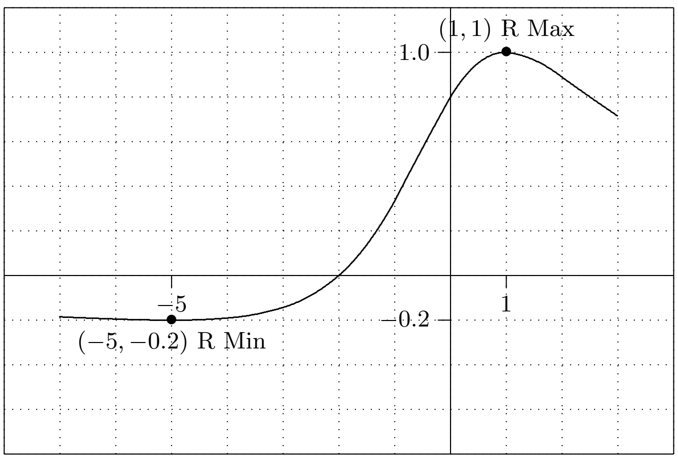
		R Min		R Max	
	$--$	0	$+++$	0	$---$
f'					
	Decr	-5	Incr	1	Decr

To do this which theorem did you use? Increasing/Decreasing Test

b) Determine which critical points are local maxima, minima, and which are not extreme and mark this on your number line. (See above.)

To do this which theorem did you use? First Derivative Test

c)



At the critical numbers:
$f(-5) = -\frac{1}{5} = -0.2$ and
$f(1) = 1$.
Now plot these points and use the increasing/decreasing information from above.

d) Now using the information in parts (a), (b), and (c). make a sketch of the graph of f *without plotting any more individual points.* The shape of the graph should come from the information in parts (a), (b), and (c).

e) From part (b), the only critical point of $f(x)$ in $[-2, 2]$ is $x = 1$.

- At the critical point: $f(1) = 1$
- and at the endpoints: $f(-2) = 0$
- and $f(2) = \frac{8}{9}$.
- Abs max is 1 at $x = 1$;
- Abs min is 0 at $x = -2$.

Which theorem did you use? Closed Interval Theorem