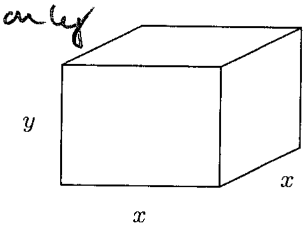
Hand In. Name: Answers 32

Do WeBWorK Set Day 32 due Tuesday night.

1. (Similar to WeBWorK Set Day 32 #1). A small box with a **square base** and **no top** is to hold a volume of 256 cc. What dimensions for the box will minimize the materials for the 4 sides and bottom? Justify your answer. Solution: Using the guidelines



a) Optimize (Minimize?, Maximize?):

4 sides, bottom only

Minimize Surface Area $= S = 4xy + x^2$

b) Constraint: Subject to: Volume $= x^2 y = 256$

c) Eliminate (if necessary): $y = \frac{256}{x^2}$, $x > 0$ or $(0, \infty)$

Any $x > 0$ means we can solve for y

d) Rewrite: $S = 4x\left(\frac{256}{x^2}\right) + x^2 = \frac{1024}{x} + x^2$

Domain: $(0, \infty)$

e) Solve and **Justify:**

$$S' = -\frac{1024}{x^2} + 2x = 0$$

$$2x = \frac{1024}{x^2}$$

$$2x^3 = 1024$$

$$x^3 = 512$$

$$\boxed{x = 8}$$

Justify: USE SCPT. There's only 1 critical point, $x = 8$ in $(0,\infty)$. By the first Derivative test f': $- - -$ (R Min at 8) $+ + +$, on $(0, \infty)$

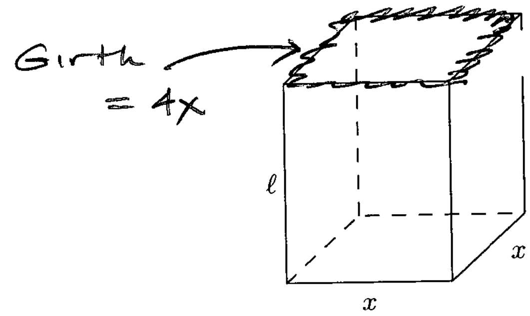
there's a relative min @ $x = 8$ $\therefore$ By SCPT there's an Absolute Min @ $x = 8$ cm

$y = \frac{256}{8^2} = 4$ cm

2. (WeBWorK Set Day 32 #4) US Postal regulations `usps.com/consumers/domestic.htm` state: "Priority Mail is used for documents, gifts, and merchandise. The maximum size is 108 inches or less in combined length (ℓ) and distance around the thickest part (Girth, G)." Find the dimensions of the box with square base of largest volume that can be sent Priority Mail. Justify your answer. Solution: Using the guidelines

a) Optimize (Minimize?, Maximize?):

Maximize $\text{Vol} = x^2 \ell$



b) The Girth is the *perimeter around the square end*, not the area of the square end (see figure). So

Subject to Girth + length = 108. So $108 = \underline{4x} + \ell$

c) Eliminate ℓ. $\ell = 108 - 4x$ ← ℓ and x cannot be negative. smallest $x = 0$, largest $x = 27$

d) Rewrite: $V = x^2(108 - 4x)$

$= 108x^2 - 4x^3$

Domain: $[0, 27]$

↑ closed... use CIT

e) Solve and **Justify:**

$V' = 216x - 12x^2 = -12x(x - 18) = 0$; $x = 0, 18$

The domain is a closed interval. Use CIT to find the Abs Max.

Check Critical pts: $V(18) = 18^2(108 - 72) = 18^2 \cdot 36 = 11{,}644 \text{ in}^3$ ← Abs Max

Endpts: $V(0) = 0$

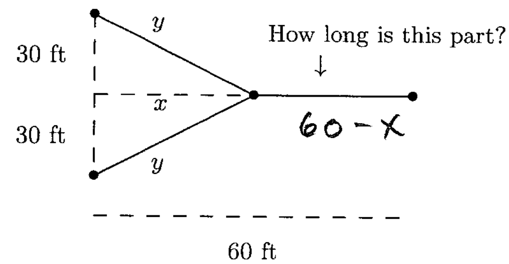
$V(27) = 27^2(108 - 4 \cdot 27) = 0$

Abs Max Vol: $11{,}644 \text{ in}^3$ by CIT

Dimensions: $x = 18$ in

$\ell = 108 - 4(18) = 36$ in

3. A cable tv guy has to run cable from where it enters the house to two separate tv's as shown. At what point x should he split the cable feed to minimize the amount of wire used? (See figure. Note the dashed line labeled 60ft. What does that make the length of the unlabeled edge?) Solution: Using the guidelines



a) Optimize (Minimize?, Maximize?):

Minimize $C = (60 - x) + 2y$

b) Constraint: Subject to: $y^2 = x^2 + 30^2$

$y = \sqrt{x^2 + 900}$

Smallest $x = 0$

Largest $x = 60$

c) Eliminate (if necessary):

$y = \sqrt{x^2 + 900}$

d) Rewrite:

$C = 60 - x + 2\sqrt{x^2 + 900}$

Domain: $[0, 60]$ Closed interval use CIT later

e) Solve and **Justify:**

$c' = -1 + 2(\tfrac{1}{2})(x^2 + 900)^{-1/2}(2x)$

$c' = -1 + \dfrac{2x}{\sqrt{x^2+900}} = 0$

So $\dfrac{2x}{\sqrt{x^2+900}} = 1$, or $2x = \sqrt{x^2+900}$ Square both sides

$4x^2 = x^2 + 900$

$3x^2 = 900$

$x^2 = 300 \Rightarrow x = +\sqrt{300}, \cancel{-\sqrt{300}}$ not in interval

Use CIT to find Abs Min.

Check @ critical point(s): $C(\sqrt{300}) = 60 - 2\sqrt{300 + 900} \approx 111.96$ Abs Min

End pts: $C(0) = 60 - 0 + 2\sqrt{900} = 120$

$C(60) = 60 - 60 + 2\sqrt{3600 + 900} \approx 134.2$

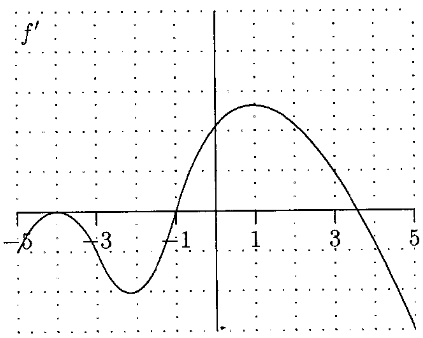
By CIT: Abs Min @ $x = \sqrt{300} = 10\sqrt{3}$ ft

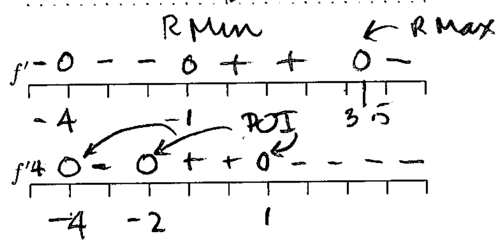
4. Below is the graph of $f'(x)$, **not** $f(x)$.

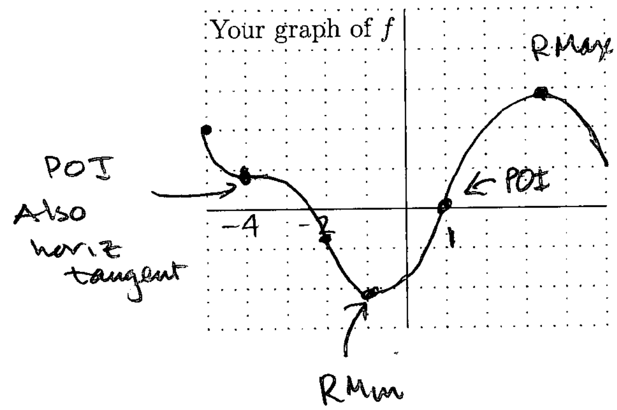
a) Translate this graphical information into number line information for f' and f''. For example, where f' is above the x-axis, f' is positive. Where the graph of f' is decreasing, that's where f'' is negative, where f' has a horizontal tangent, that's where $f'' = 0$. Here are some specific examples

- At $x = 3.5$: Note $f'(3.5) = 0$ (the graph crosses the x-axis and $f''(3.5)$ is negative because $f'(x)$ is decreasing there Mark each on the corresponding number line.
- At $x = 1$: Note $f'(1)$ is positive (the graph is above the the x-axis and $f''(1) = 0$ because $f'(x)$ has a horizontal tangent there. Mark each on the corresponding number line.
- Continue in the same manner. You should be able to fill in each number line where it is 0, +, and −.

b) Based on the number line information (relative extrema, inflections, etc.), draw a possible graph of the original function $y = f(x)$. Start your graph at the indicated point •.







5. Extra Credit: Like WeBWorK Set Day 32 #5: The manager of a large apartment complex knows from experience that 80 units will be occupied if the rent is 372 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 6 dollar increase in rent. Similarly, one additional unit will be occupied for each 6 dollar decrease in rent. What rent should the manager charge to maximize revenue?

Maximize Revenue $= (372+6x)\cdot(80-x)$

↑ price ↑ ↑ # units rented

↑ both must be non-negative (or positive)

Domain: So:

$372 + 6x \geq 0$, $6x \geq -372$, $x \geq -62$

And $80 - x \geq 0$, $80 \geq x$

Domain: $[-62, 80]$ ← use CIT

Solve So

$R = (372+6x)(80-x)$ on $[-62, 80]$

$R' = 6(80-x) + (372+6x)(-1) = 480 - 6x - 372 - 6x = 0$

$108 = 12x$ or $x = 9$

use CIT. (or use SCPT)

Check @ Crit Pt: $R(9) = (372 + 6\cdot 9)(80-9) = \$30{,}246$

@ End pts: $R(-62) = (372 - 6\cdot 62)(80+62) = 0$

$R(80) = (372 + 6\cdot 80)(80-80) = 0$

AbsMax when $x = 9$ price = \$326