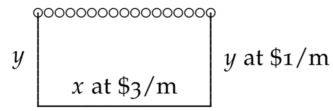
Math 130, Day 33 Answers

1. Maximize: Area $A = xy$



Constraint: Cost $C = 3x + 2y = 600$

Eliminate: $y = 300 - \frac{3}{2}x$ Smallest $x = 0$, largest $x = 200$ since $3(200) = 600$

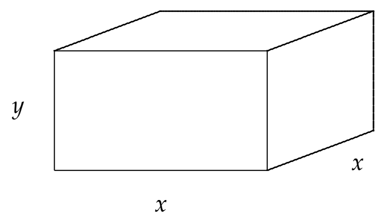
Substitute: $A = x(300 - \frac{3}{2}x) = 300x - \frac{3}{2}x^2$ Domain: $[0, 200]$

Differentiate: $A' = 300 - 3x = 0 \Rightarrow x = 100$

Justify: Since the interval is closed, we can use the CIT. Evaluate A at the critical and end points.
At the Critical point: $A(100) = 100(300 - \frac{3}{2}100) = 15,000$ (Abs Max).
At the End points: $A(0) = 0$, $A(200) = 0$.
So by the CIT, the Abs Max of $15,000$ square meters occurs at $x = 100$ m and $y = 300 - \frac{3}{2}100 = 150$ m.

2.

2 layers on top and bottom



Minimize: Materials $S = \overbrace{4x^2 + 4xy}^{\text{2 tops, 2 bottoms + 4 sides}}$

Constraint: Volume $V = x^2y = 16000$

Eliminate: $y = \frac{16000}{x^2}$ y can be any value larger than 0

Substitute: $S = 4x^2 + \frac{64000}{x}$ Domain: $(0, \infty)$

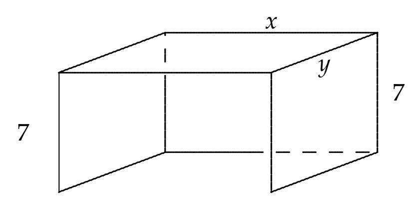
Differentiate: $S' = 8x - \frac{64000}{x^2} = 0 \Rightarrow 8x^3 = 64000$ So $x^3 = 8000$ so $x = 20$

Justify: There is only one CP, so use the SCPT. Classify the CP with the First Derivative Test.

$$S' \quad \underset{0}{\circ}\; - - - \;\; \underset{20}{\text{R Min}} \;\; + + +$$

There is a Relative Min at $x = 20$ which must be an Abs Min by SCPT. $y = 40$.

3.



No floor, no front

Minimize: Materials $S = \overbrace{2(7y) + 7x + xy}^{\text{2 sides + back + top}}$

Constraint: Volume $V = 7xy = 686$

Eliminate: $x = \frac{98}{y}$ y can be any value larger than 0

Substitute: $S = 14y + 7(\frac{98}{y}) + (\frac{98}{y})y = 14y + \frac{686}{y} + 98$ Domain: $(0, \infty)$. Think SCPT

Differentiate: $S' = 14 - \frac{686}{y^2} = 0 \Rightarrow 14y^2 = 686 \Rightarrow y^2 = 49$ $y = 7$ but not -7

Justify: There is only one CP, so use the SCPT. Classify the CP with the First Derivative Test.

$$S' \quad \underset{0}{\circ}\; - - - \;\; \underset{7}{\text{R Min}} \;\; + + +$$

There is a Relative Min at $y = 7$ feet. By the SCPT, this is an Abs Min.
Dimensions: $y = 7$ and $x = \frac{98}{7} = 14$ feet (and height 7).

4.

Maximize:	Sum of squares $S = x^2 + y^2$	
Constraint:	$x + y = 4$	$x \geq 0$ and $y \geq 0$ (both positive)
Eliminate:	$y = 4 - x$	Need: $x \geq 0$ and $x \leq 4$
Substitute:	$S = x^2 + (4-x)^2 = 2x^2 - 8x + 16$	Domain: $[0, 4]$
Differentiate:	$S' = 4x - 8 = 0 \Rightarrow x = 2$	
Justify:	The interval is closed	Use the CIT
Check CP: $S(2) = 8$	Check Endpts: $S(0) = 16$, $S(4) = 16$	
By CIT, Abs max of 16 at	either $x = 0,\ y = 4$	or $x = 4,\ y = 0$.

5. (*a*) $\displaystyle \lim_{x\to+\infty} \frac{3-2x^2}{3x^3-1} \overset{\text{HP}}{=} \lim_{x\to+\infty} \frac{-2x^2}{3x^3} = \lim_{x\to+\infty} \frac{-2}{3x} = 0$

(*b*) $\displaystyle \lim_{x\to+\infty} \frac{3-2x}{3x-1} \overset{\text{HP}}{=} \lim_{x\to+\infty} \frac{-2x}{3x} = -\frac{2}{3}$

(*c*) $\displaystyle \lim_{x\to-\infty} \frac{3-2x^2}{3x-1} \overset{\text{HP}}{=} \lim_{x\to-\infty} \frac{-2x^2}{3x} = \lim_{x\to-\infty} \frac{-2x}{3} = +\infty$

(*d*) $\displaystyle \lim_{x\to0^-} \frac{\overbrace{x^2+x-2}^{-2}}{\underbrace{x^2}_{0^+}} = -\infty$

(*e*) $\displaystyle \lim_{x\to1^-} \frac{x^2+x}{x^2-1} = \lim_{x\to1^-} \frac{x(x+1)}{(x-1)(x+1)} = \lim_{x\to1^-} \frac{\overbrace{x}^{1}}{\underbrace{x-1}_{0^-}} = -\infty$