

Math 130 Dy 37, Hand In. Name: Answers

0. Do WeBWorK Set Day 37, due Thursday night. Finish WeBWorK Set Day 35-36 tonight.

1. Use L'Hopital's rule, if appropriate, to determine each of these limits after determining the indeterminate form of the limit. Show your work.

a)  $\lim_{x \rightarrow 0} \frac{\tan x}{\sin x} = \frac{\overset{0}{\cancel{\tan x}}}{\cancel{\sin x}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow 0} \frac{\sec^2 x}{\cos x} = \frac{\overset{1}{\cancel{\sec^2 x}}}{\cancel{\cos x}} \rightarrow 1$

b)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x} = \frac{\overset{0}{\cancel{\sin 5x}}}{\cancel{\sin 2x}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow 0} \frac{5 \cos 5x}{2 \cos 2x} = \frac{\overset{5}{\cancel{5 \cos 5x}}}{2 \cancel{\cos 2x}} \rightarrow \frac{5}{2}$

c)  $\lim_{x \rightarrow +\infty} \frac{x^2}{e^{2x}} = \frac{\overset{\infty}{\cancel{x^2}}}{\cancel{e^{2x}}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow \infty} \frac{2x}{2e^{2x}} = \frac{\overset{\infty}{\cancel{2x}}}{\cancel{2e^{2x}}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow \infty} \frac{2}{4e^{2x}} = \frac{\overset{2}{\cancel{2}}}{4 \cancel{e^{2x}}} \rightarrow 0$

d)  $\lim_{x \rightarrow +\infty} \frac{\ln 2x}{x^2} = \frac{\overset{\infty}{\cancel{\ln 2x}}}{\cancel{x^2}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow \infty} \frac{\frac{2}{2x}}{\cancel{2x}} = \lim_{x \rightarrow 0} \frac{2}{2x} \cdot \frac{1}{2x} = \lim_{x \rightarrow 0} \frac{\frac{1}{2x^2}}{\cancel{2x}} \stackrel{r^1}{\rightarrow} 0$

or  $\lim_{x \rightarrow \infty} \frac{\frac{2}{2x}}{\cancel{2x}} \stackrel{r^2}{\rightarrow} 0$

e)  $\lim_{x \rightarrow 1} \frac{6^x + 4^x - 10}{e^{x-1} - 1} = \frac{\overset{6+4-10=0}{\cancel{6^x + 4^x - 10}}}{\cancel{e^{x-1} - 1}} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow 1} \frac{6^x \ln 6 + 4^x \ln 4}{e^{x-1}} = \frac{6 \ln 6 + 4 \ln 4}{e^0} = 6 \ln 6 + 4 \ln 4$

f) If we get this far:  $\lim_{x \rightarrow 0^+} x^2 \ln x^2 = \frac{\overset{0}{\cancel{x^2 \ln x^2}}}{\cancel{x^2} \rightarrow 0^+} \stackrel{-\infty}{\rightarrow}$  Recip  $\frac{\ln x^2}{\frac{1}{x^2} \rightarrow \infty} \stackrel{-\infty}{\rightarrow} \stackrel{L'H}{\rightarrow} \lim_{x \rightarrow 0^+} \frac{\frac{2x}{x^2}}{-\frac{2}{x^3}} = \lim_{x \rightarrow 0^+} \frac{\frac{2}{x}}{-\frac{2}{x^3}}$

$$= \lim_{x \rightarrow 0^+} \frac{2}{x} \cdot \frac{x^3}{-2} = \lim_{x \rightarrow 0^+} -x^2 = 0$$

OVER

2. Draw a detailed graph of  $f(x) = \frac{x}{e^x}$ . Include all extrema, inflections, and asymptotes (both VA and HA, if any) and any appropriate 'end behavior.' Indicate any intercepts as well. Show all work. This is the same as WeBWorK Problem 1 on Set Day 37 which you should do first! Be especially careful evaluating  $\lim_{x \rightarrow -\infty} \frac{x}{e^x}$ . It is NOT of the indeterminate form  $\frac{\infty}{\infty}$ . What is it?

Domain:  $f(x) = \frac{x}{e^x}$ .  $e^x \neq 0$  so domain is  $(-\infty, \infty)$ , No VA

$$\text{HA: } \lim_{x \rightarrow \infty} \frac{x}{e^x} \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0 \quad y=0 \text{ HA}$$

$$\lim_{x \rightarrow -\infty} \frac{x}{e^{-x}} = -\infty \quad \leftarrow \text{Careful} \quad \text{R Max}$$

$$\text{CPS: } f'(x) = \frac{e^x - xe^x}{(e^x)^2} = \frac{1-x}{e^x} = 0 \text{ @ } x=1 \quad f' \begin{array}{c} + + + \\ \hline \text{inc} & | & \text{dec} \end{array}$$

Possible POIs

$$f''(x) = \frac{-e^x - (1-x)e^x}{(e^x)^2} = \frac{(-1+1+x)}{e^x} = \frac{-2+x}{e^x} = 0 \text{ @ } x=2$$

$$f'' \begin{array}{c} - - - \\ \hline \text{cur DN} & | & \begin{array}{c} \text{POI} \\ 0 \end{array} & + + + \\ \hline & 2 & \text{cur up} \end{array}$$

Key Points

$$f(1) = \frac{1}{e} \approx .367$$

$$f(2) = \frac{2}{e^2} \approx .271$$

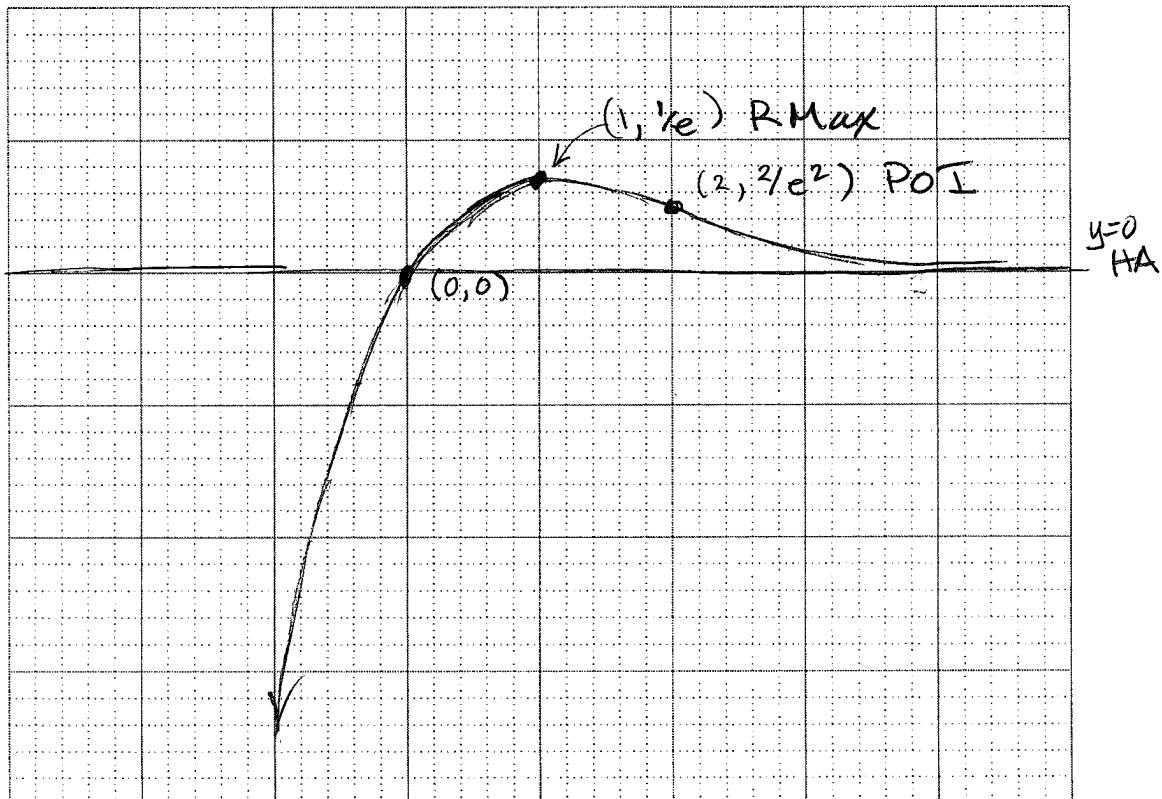
y intercept:  $(x=0)$

$$f(0) = 0$$

x intercept ( $y=0$ )

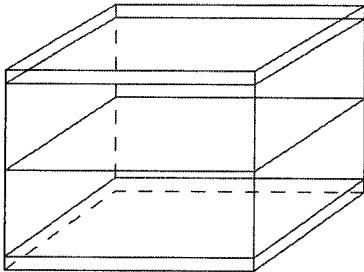
$$f(x) = \frac{x}{e^x} = 0$$

only at  $x=0$



## Optional: Max-Min and Related Rate Re-dos. Answers

1. A small cardboard box is to contain a volume of 2,500 cm<sup>3</sup>. The box has a **square base** and is constructed with **two layers** of cardboard on the top and bottom. Inside it has an additional horizontal cardboard divider as shown [to make two compartments]. Find the dimensions of the box that minimize the amount of materials used for the sides, top, and bottom. Carefully justify your answer.



$$\text{Minimize } A = 5x^2 + 4xh \text{ (2 tops, 2 bottoms, divider, & 4 sides)}$$

$$\text{Constraint: } V = x^2h = 2500 \text{ in}^3$$

$$\text{Eliminate: } h = \frac{2500}{x^2} \text{ on } (0, \infty).$$

$$\text{So: } A = 5x^2 + 4x \cdot \frac{2500}{x^2} = 2x^2 + \frac{10,000}{x} \text{ on } (0, \infty). \text{ Therefore:}$$

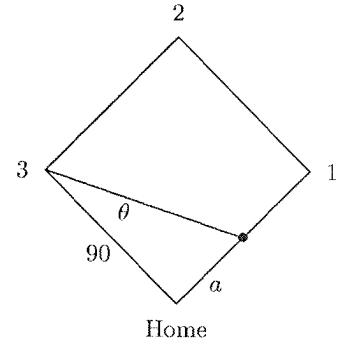
$$A' = 10x - \frac{10,000}{x^2} = 0 \Rightarrow 10x = \frac{10,000}{x^2} \Rightarrow x^3 = 1000 \Rightarrow x = 10.$$

Using the First Derivative Test to classify the CP, there's a local min at  $x = 10$

|      |       |    |     |
|------|-------|----|-----|
| $f'$ | ----- | 0  | +++ |
|      | 0     | 10 |     |

Since there's a single critical point, by the SCPT, the rel min at  $x = 10$  is an abs min.  $h = 2500/(10)^2 = 25$ .

2. A baseball diamond is a square with 90 ft sides. Derek Jeter hits the ball and runs towards first base at a speed of 24 ft/s. How is the angle  $\theta$  changing at this moment?



Solution: Given  $\frac{da}{dt} = 24$  ft/s.

$$\text{Find } \left. \frac{d\theta}{dt} \right|_{a=45}.$$

Relation (use the constant side!):  $\tan \theta = \frac{a}{90}$ , so  $\theta = \arctan(\frac{a}{90})$ .

Rate-ify:

$$\frac{d\theta}{dt} = \frac{1}{1 + (\frac{a}{90})^2} \cdot \frac{1}{90} \frac{da}{dt} = \frac{90}{90^2 + a^2} \frac{da}{dt}.$$

Evaluate. When  $a = 45$ ,

$$\left. \frac{d\theta}{dt} \right|_{a=45} = \frac{90}{90^2 + 45^2} (24) = 0.213 \text{ rad/s.}$$

a) Another method: Find  $\left. \frac{d\theta}{dt} \right|_{a=45}$ .

$$\text{Relation: } \tan \theta = \frac{a}{90}.$$

$$\text{Rate-ify: } \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{90} \frac{da}{dt} \text{ or } \frac{d\theta}{dt} = \frac{1}{90 \sec^2 \theta} \frac{da}{dt} = \frac{\cos^2 \theta}{90} \frac{da}{dt}.$$

Evaluate: Use the triangle: When  $a = 45$ , from part (a) we have  $c = 45\sqrt{5}$ , so  $\cos^2 \theta = (\frac{90}{c})^2 = (\frac{90}{45\sqrt{5}})^2 = (\frac{2}{\sqrt{5}})^2 = \frac{4}{5} = 0.8$ . So

$$\left. \frac{d\theta}{dt} \right|_{a=45} = \frac{\cos^2 \theta}{90} \frac{da}{dt} = \frac{0.8}{90} (24) = 0.213 \text{ rad/s.}$$