Math 130, Day 39. Answers

1. Eight from your text: Do page 328–29. Use the rules on pages 322 and 324 or below. **Check your answers by differentiating.** NOTE: Write out the original question, not just the answer.

a) #24

power rule

$$\int (3u^{-2} - 4u^2 + 1)\,du = \frac{3u^{-1}}{-1} - \frac{4u^3}{3} + u + C = -3u^{-1} - \frac{4}{3}u^3 + u + C$$

b) #26

Rewrite

$$\int \left(\frac{5}{t^2} + 4t^2\right)dt = \int 5t^{-2} + 4t^2 = \frac{5t^{-1}}{-1} + \frac{4t^3}{3} + C = -5t^{-1} + \frac{4}{3}t^3 + C$$

c) #28 (multiply first)

$$\int 5m(12m^3 - 10m)\,dm = \int 60m^4 - 50m^2\,dm = \frac{60m^5}{5} - \frac{50m^3}{3} + C$$

$$= 12m^5 - \frac{50m^3}{3} + C$$

d) #30

$$\int 6\sqrt[3]{x}\,dx = \int 6x^{1/3}\,dx = \frac{6x^{4/3}}{4/3} + C = \frac{9}{2}x^{4/3} + C$$

e) #32

$$\int (4z^{1/3} - z^{-1/3})\,dz = \frac{4z^{4/3}}{4/3} - \frac{z^{2/3}}{2/3} + C = 3z^{4/3} - \frac{3}{2}z^{2/3} + C$$

f) #34 (rewrite as a fractional power)

$$\int \sqrt[5]{r^2}\,dr = \int r^{2/5}\,dr = \frac{r^{7/5}}{7/5} + C = 5/7\, r^{7/5} + C$$

g) #36 (divide first)

$$\int \frac{12t^8 - t}{t^3}\,dt = \int 12t^5 - t^{-2}\,dt = \frac{12t^6}{6} - \frac{t^{-1}}{-1} + C = 2t^6 + t^{-1} + C$$

h) #48.

$$\int e^{2t} + 2\sqrt{t}\,dt = \int e^{2t} + 2t^{1/2}\,dt = \frac{1}{2}e^{2t} + \frac{2t^{3/2}}{3/2} + C = \frac{1}{2}e^{2t} + \frac{4}{3}t^{3/2} + C$$

Should check all by taking derivatives!

2. Given $\frac{dx}{dt} = -10$ m/s (distance to finish line is decreasing!).
Find $\left.\frac{d\theta}{dt}\right|_{x=15}$. Relation: $\tan\theta = \frac{x}{5}$.
Rate-ify: $\sec^2\theta\frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$ or $\frac{d\theta}{dt} = \frac{1}{5\sec^2\theta}\frac{dx}{dt} = \frac{\cos^2\theta}{5}\frac{dx}{dt}$.
Evaluate: Use the triangle.
When $x = 15$, the hypotenuse is $\sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$. So $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{5\sqrt{10}} = \frac{1}{\sqrt{10}}$. So

$$\left.\frac{d\theta}{dt}\right|_{x=5} = \frac{\cos^2\theta}{5}\frac{dx}{dt} = \frac{\left(\frac{1}{\sqrt{10}}\right)^2}{5}(-10) = \frac{\frac{1}{10}}{5}(-10) = -0.2\,\text{rad/s}.$$