Math 130, Day 4. Hand In Next Time. Name: ______________________

Remember: Do WeBWorK Set Day04. Start now!

1. These limits are straightforward. Note which limit rules you apply. **Make sure to write out the question**, not just the answer. State which limit law(s) you apply at each step (as we did in class and as in Example 2, p. 70.

a) Page 76 #22

$$\lim_{x\to1}\frac{f(x)}{g(x)-h(x)} \overset{\text{quotient}}{=} \frac{\lim_{x\to1} f(x)}{\lim_{x\to1} g(x)-h(x)} \overset{\text{Difference}}{=} \frac{\lim_{x\to1} f(x)}{\lim_{x\to1} g(x)-\lim_{x\to1} h(x)} = \frac{8}{3-2} = 8$$

b) Page 76 #24

$$\lim_{x\to1}\sqrt[3]{f(x)g(x)+3} \overset{\text{Frac power}}{=} \sqrt[3]{\lim_{x\to1} f(x)g(x)+3} \overset{\text{sum, product}}{=} \sqrt[3]{\lim_{x\to1} f(x)\cdot\lim_{x\to1} g(x)+\lim_{x\to1} 3}$$

$$\overset{\text{constant}}{=} \sqrt[3]{8\cdot3+3} = \sqrt[3]{27} = 3$$

c) Page 76 #30

$$\lim_{x\to2}(x^2-x)^5 \overset{\text{power}}{=} \left(\lim_{x\to2} x^2-x\right)^5 \overset{\text{polynomial}}{=} (2^2-2)^5 = 2^5 = 32$$

2. For these next problems, you cannot immediately apply a limit law since the form is indeterminate: $\frac{0}{0}$. Carry out appropriate algebraic simplification and then evaluate the limit. Write out the question and **be sure to use proper notation** (limit symbols, equal signs). You will need to simplify before evaluating the limit.

a) Page 77 #40

$$\lim_{x\to3}\frac{x^2-2x-3}{x-3} = \lim_{x\to3}\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = \lim_{x\to3} x+1 \overset{\text{poly}}{=} 4$$

b) Page 77 #46

$$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{\frac{5-(5+h)}{(5+h)(5)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{(5+h)(5)}}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{(5+h)(5)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{(5+h)(5)}$$

evaluate (Rational) $= \frac{-1}{(5+0)(5)} = -\frac{1}{25}$

3. Average Velocity Review. Remember that m_{sec} is the same as average velocity. For example, if we the position of an object is given by $f(x)$ and we want the average velocity on the general interval $[a, x]$, then we use the difference quotient

$$m_{sec} = \text{Ave Velocity} = \frac{f(x) - f(a)}{x - a}.$$

Let $f(x) = x^2 + 5x + 1$ represent the position of an object. Determine the average velocity on the general interval $[2, x]$ by simplifying the difference quotient above by using $a = 2$. If necessary, review the online Answers to the Day 2 problems.

$f(2) = 4 + 10 + 1$

$$m_{sec} = \text{Ave Vel} = \frac{f(x) - f(2)}{x-2} = \frac{x^2 + 5x + 1 - (15)}{x-2}$$

$$= \frac{x^2 + 5x - 14}{x-2} = \frac{\cancel{(x-2)}(x+7)}{\cancel{x-2}} = x + 7 \qquad (x \neq 2)$$

4. Tammy has been asked to determine the instantaneous velocity at time $x = 4$ of an object that has position $f(x) = x^2 - 7x$. She knows that instantaneous velocity at $x = 4$ is a limit of average velocity as $x \to 4$:

$$\text{Inst Vel} = \lim_{x\to 4} \text{Ave Velocity} = \lim_{x\to 4} \frac{f(x) - f(4)}{x - 4}.$$

a) Why can't Tammy just plug $x = 4$ into this expression?

She'd get $\frac{f(4)-f(4)}{4-4} = \frac{0}{0}$, undefined!

b) Help Tammy determine the instantaneous velocity at $x = 4$ by using some algebra as in the previous problem and then evaluating the limit. Use proper notation (limit symbols, equal signs).

$f(4) = 16 - 28$

$$\text{Inst Vel} = \lim_{x\to 4} \frac{f(x)-f(4)}{x-4} = \lim_{x\to 4} \frac{x^2-7x-(-12)}{x-4}$$

$$= \lim_{x\to 4} \frac{x^2-7x+12}{x-4} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x-3)}{\cancel{x-4}}$$

poly

$$= \lim_{x\to 4} x-3 = 1$$

5. Prove that you are an average/instantaneous velocity pro. Let $s(t) = \frac{2}{t}$ represent the position of an object at time t. Find the instantaneous velocity at $t = 8$ as a limit of average velocity using the process in the previous problem. Use proper notation (limit symbols, equal signs). Review the algebra in the answer to Day 2, #6 (b) on today's handout.

$$\text{Inst. Vel} = \lim_{t\to 8} \frac{s(t)-s(8)}{t-8} = \lim_{t\to 8} \frac{\frac{2}{t}-\frac{2}{8}}{t-8} = \lim_{t\to 8} \frac{\frac{16-2t}{8t}}{t-8}$$

$$= \lim_{t\to 8} \frac{\frac{-2(t-8)}{8t}}{t-8} = \lim_{t\to 8} \frac{-2\cancel{(t-8)}}{8t} \cdot \frac{1}{\cancel{t-8}}$$

Rational

$$= \lim_{t\to 8} \frac{-2}{8t} = \frac{-2}{8(8)} = \frac{-1}{32}$$

6. Which of these are best described as polynomials (P)? As rational (R)? Neither (N)?

N **a)** $\sqrt{x^2+12x+100}$ R **b)** $\dfrac{2x^{10}-4x}{x^2+3x+1}$ N **c)** $\dfrac{x}{x^{1/2}+1}$ N **d)** $2x^5-3x^{2/3}+1$

R **e)** $\dfrac{9}{6x^2+2}$ **f)** $\sin(x^2+3x+1)$ P **g)** x^2-6x+1 **h)** $\sqrt{2}$ P

7. Remember how functions work. Let $f(x)=x^2+5x+1$. To evaluate $f(2+h)$ we simply substitute $2+h$ for x:

$$f(2+h)=(2+h)^2+5(2+h)+1=(4+4h+h^2)+(10+5h)+1=h^2+9h+15.$$

Be careful to square properly and to multiply through by constants as needed. (Also see the answer to Day 2, #3 (d) on today's handout.)

a) Let $f(x)=x^2-3x$. Evaluate $\lim\limits_{h\to 0}\dfrac{f(3+h)-f(3)}{h}$. This really simplifies. Be sure to use proper notation (limit symbols, equal signs). Only evaluate the limit after you have simplified.

$$\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{(3+h)^2-3(3+h)-[9-9]}{h}=\lim_{h\to 0}\frac{9+6h+h^2-9-3h}{h}$$

$$=\lim_{h\to 0}\frac{3h+h^2}{h}=\lim_{h\to 0}3+h=3$$

b) Optional Bonus. Let $f(x)=\sqrt{x}$. Evaluate $\lim\limits_{h\to 0}\dfrac{f(4+h)-f(4)}{h}$. You will need to use conjugates. You can read about conjugates in today's online Class Notes for Day 04.

conjugate

$$\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}\cdot\frac{\sqrt{4+h}+\sqrt{4}}{\sqrt{4+h}+\sqrt{4}}$$

$$=\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+\sqrt{4})}=\lim_{h\to 0}\frac{\cancel{h}\;1}{\cancel{h}(\sqrt{4+h}+\sqrt{4})}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+\sqrt{4}}=\frac{1}{\sqrt{4}+\sqrt{4}}=\boxed{\frac{1}{4}}$$