Math 130 Homework: Day 2

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Reading, Practice, Review

1. Reading/Review: Review the idea of limits and the tangent line problem in Section 2.1. Begin your study of **Limits** in Section 2.2. **Concentrate**; there is a lot of material there and it requires careful attention.

a) Practice page 59: Concept Review #1, 3, 5. Basic skills: #7, 9, 15, 19. #7 and 9 are related to a hand-in problem below.

2. Practice. Review of **logs, exponentials, and inverse functions**, see Chapter 1.3. Try pages 35–36: #5, 6, 11, 17, 23, 31 (see Example 4), 39, 41, and 43. (Answer to #6: $y = \frac{x}{3} + \frac{4}{3}$.)

3. **Average and Instantaneous Velocity**. Consider the position function $s(t) = -16t^2 + 128t$ (see Example 1 and 2 in Section 2.1 and Exercises #9 and 11 on page 59).

a) Find and simplify the expression for m_{sec} =average velocity on the interval $[2, t]$. Hint: Factor out -16 from the difference quotient.

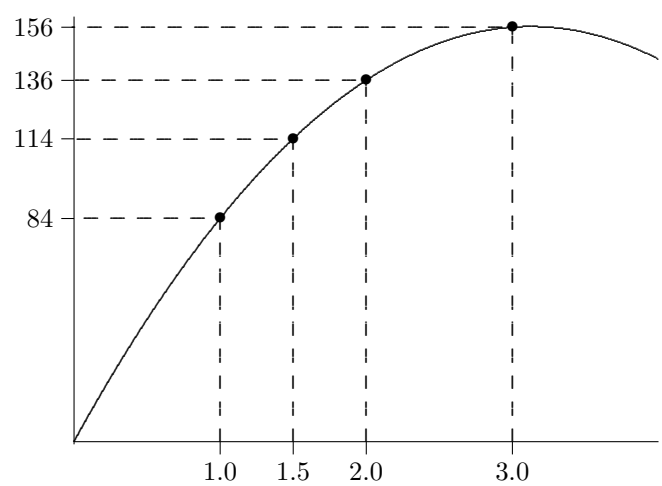
b) Use your formula for the difference quotient from part (a) to quickly evaluate average velocities on the given intervals. See the Classwork example. Also, see the online class notes for Day 2.

$[2, t]$	m_{sec} = ave vel on $[2, t]$
$[2, 3]$	
$[2, 2.5]$	
$[2, 2.1]$	
$[2, 2.01]$	
$[2, 2.001]$	
$[2, 2.0001]$	

c) Make a conjecture (educated guess) about the instantaneous velocity right at $t = 2$.

4. **Average Velocity**. The graph below gives the position of an object moving along a line at time t, over a 2-second interval. Find the average velocity of the object (use the difference quotient) for the given intervals.

a) $[1, 3]$ **b)** $[1, 2]$ **c)** $[1, 1.5]$



Classwork: Instantaneous Rates and The Tangent Slope Problem

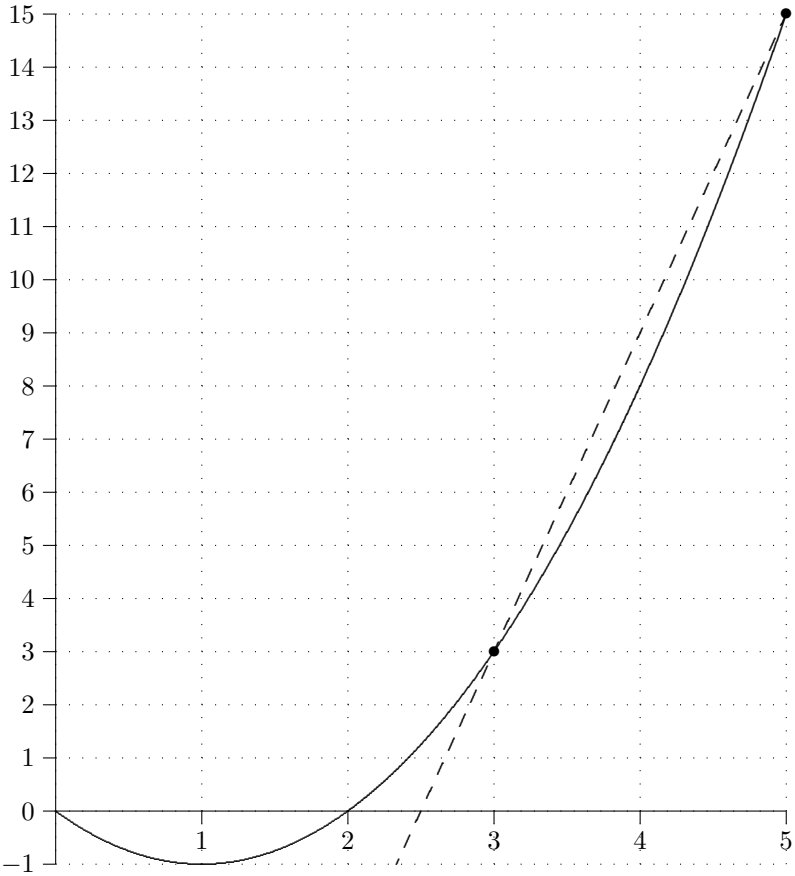
General Setup: Given a function $y = f(x)$ with a in the domain of f. The **secant line** through the points $(a, f(a))$ and $(x, f(x))$ on the graph of f is the **difference quotient**

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(a)}{x - a}.$$

Note: If f represents position (height or distance) and x represents time, then

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a} = \frac{\Delta\text{position}}{\Delta\text{time}} = \text{Average velocity during time interval } [a, x].$$

Consider the function $f(x) = x^2 - 2x$ near $a = 3$ which is graphed below.



$[3, x]$	m_{sec} = ave vel on $[3, x]$
$[3, 5]$	
$[3, 4]$	
$[3, 3.5]$	
$[3, 3.1]$	
$[3, 3.01]$	
$[3, 3.001]$	
$[3, 3.0001]$	
$[2.9999, 3]$	
$[2.999, 3]$	
$[3, 2.99, 3]$	
$[2.9, 3]$	
$[2.5, 3]$	
$[2.0, 3]$	

First find m_{sec} (average velocity) on the interval $[3, 5]$.

$$\text{Average Velocity} = m_{\text{sec}} = \frac{f(5) - f(3)}{5 - 3} = \frac{(\qquad\qquad) - (9 - 6)}{2} = \frac{\qquad -3}{2} = 6 \text{ (m/s)}. \tag{1}$$

The secant line through $(3, f(3))$ and $(5, f(5))$ with slope 6 is drawn in the graph. This slope is a rough approximation of the slope of the curve right at $(3, f(3))$.

Let's try for a better approximation: Choose x closer to $a = 3$. Try $x = 4$. Draw the secant.

$$\text{Average Velocity} = m_{\text{sec}} = \frac{f(4) - f(3)}{4 - 3} = \frac{(\qquad\qquad) - 3}{1} = \qquad\qquad \text{(m/s)}. \tag{2}$$

Is this second approximation closer to the slope of the curve right at $(3, f(3))$ than our first approximation?
Here's one more point: Choose x closer still to 3. Try $x = 3.5$.

$$\text{Average Velocity} = m_{\text{sec}} = \frac{f(3.5) - f(3)}{3.5 - 3} = \frac{5.25 - 3}{0.5} = \qquad\qquad \text{(m/s)} \tag{3}$$

Try it in general using just x.

$$\text{Average Velocity} = m_{\text{sec}} = \frac{f(x) - f(3)}{x - 3} = \frac{x^2 - 2x - 3}{x - 3} = \qquad\qquad \text{(m/s) } [x \neq 3] \tag{4}$$

Fill in the table! What is the instantaneous velocity right at 3? What is the slope of the curve right at $x = 3$?

Math 130 Day 02. Hand In At Class. Name: _____

0. Do WeBWorKSet Day02. Do them with your homework below. Many are similar. Today's class and Lab from Thursday should help you. Also, see the online class notes for Day 2.

1. Review: Factoring. Simplify each expression by canceling any common factors. See WeBWorK Set Day02, #1.

a) $\dfrac{x^2 - 7x + 10}{x - 5}$ for $x \neq 5$

b) $\dfrac{4x^2 - 20x - 24}{x - 6}$ for $x \neq 6$

2. If $f(x) = 2x^2 + 2x + 2$, simplify the difference quotient $\dfrac{f(x) - f(1)}{x - 1}$ (where $x \neq 1$). WeBWorK Set Day02, #2.

3. Page 59, #9. These are just basic calculations. Nothing fancy. Write out the average velocity (difference quotient) and then use a calculator to compute the result.

a) $[1, 4]$

b) $[1, 3]$

c) $[1, 2]$

d) $[1, 1 + h]$. Your answer will be a formula that contains h.

4. Average and Instantaneous Velocity. The position of an object is give by the function $f(x) = x^2 + 6x$ meters.

a) Find and simplify the expression for average velocity on the interval $[1, x]$. This is the same as $m_{\text{sec}} = \dfrac{f(x) - f(1)}{x - 1}$ (where $x \neq 1$).

b) Now use your simplified formula from part (a) to fill in the tables below with the average velocities. The answers should be VERY easy to calculate, even without a calculator. Your formula will work whether x is larger or smaller than 1.

$[1, x]$	m_{sec} = Average velolcity on $[1, x]$
$[1, 2]$	
$[1, 1.5]$	
$[1, 1.1]$	
$[1, 1.01]$	
$[1, 1.001]$	

$[x, 1]$	m_{sec} = Average velolcity on $[x, 1]$
$[0, 1]$	
$[0.5, 1]$	
$[0.9, 1]$	
$[0.99, 1]$	
$[0.999, 1]$	

c) Make a conjecture (educated guess) about the instantaneous velocity right at $x = 1$. _____

5. Average and Instantaneous Velocity. The position of an object is give by the function $f(x) = \ln x$ meters. The expression for average velocity on the interval $[1, x]$ is

$$\text{average velocity} = m_{\text{sec}} = \frac{f(x) - f(0)}{x - 0} = \frac{\ln x - \ln 1}{x - 1}, \text{ where } x \neq 0.$$

There is not much simplification that you can do other than to evaluate $\ln 1$. So this time you will need to use a calculator to evaluate the following average velocities. (Make sure it is set to **radians**.) Give your

$[1, x]$	m_{sec} = Average velolcity on $[1, x]$
$[1, 1.1]$	
$[1, 1.01]$	
$[1, 1.001]$	

Make a conjecture (educated guess) about the instantaneous velocity right at $x = 0$. _____

6. a) Finding the general formula for a secant slope. (See WeBWorK Set Day02, #2.) Let $f(x) = x^2 + x + 3$. The point $P = (4, 23)$ lies on the curve. Suppose that $Q = (x, x^2 + x + 3)$ is any other point on the graph of f. Find the slope of the secant line, m_{sec} through P and Q. Simplify your answer. It will be a function of x.

b) The algebra is a little harder in this one. Let $f(x) = \frac{1}{x}$. The point $P = (5, \frac{1}{5})$ lies on the curve. Let $Q = (x, \frac{1}{x})$ be any other point on the graph of f. Find the slope of the secant line, m_{sec}, through P and Q. Simplify your answer. It will be a function of x.

7. Five-Minute Review: Polynomial Functions (See the back of this sheet or Section 1.2, page 12, or the online notes for Day 2). A **polynomial** is a function of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a non-negative integer and the a's are real numbers. If $a_n \neq 0$, then n is the **degree** of the polynomial (largest power). Note: All the powers of x must be non-negative integers. No fractional powers, no negative powers. Circle the functions below that are polynomials? For those that are polynomials, write their **degree** beside them.

a) $-\frac{2}{3}x^5 + 3x^4 + x^2 - 11$

b) $5x^2 - x^{1/3} - 23$

c) 4

d) $6x^{-2} + 4x^{-1} + 2$

e) $\sqrt{3x^{12} + 11x^9 + 12}$

f) $\dfrac{2x^2 + x}{7x + 1}$

g) $\dfrac{1}{6x^2} + \dfrac{2}{x}$

h) $6^{1/2}x^3 - 4x + 7$

i) Look up the definition of a **rational function** on page 13. Then find a rational function in the list above.