Math 130 Homework: Day 3

Practice, Reading

Reading: Reread Chapter 2.2. Read Chapter 2.3 through page 74. This covers methods of calculating limits.

1. **a)** Try page 65ff: #1, 3 (read in text), 7, 9, 11, 13, 15 (graphing calculator or online graphing), 21, 23, 29, 37 (use the graph).

2. Practice: A calculus book is dropped from a rooftop 208 ft high on the planet Mercury. The height of the book above the ground after x seconds is given by $y = f(x) = -13x^2 + 208$ ft.
   a) Let $(x, f(x))$ be any point on the curve. Find the general formula for the average velocity between $(3, f(3))$ and $(x, f(x))$. **Simplify** your answer by factoring.
   b) Find the instantaneous velocity of the book at time $x = 3$ by evaluating the limit as $x \to 3$ of the average velocity. (Ans: -78ft/s)

3. **Hand in next class:** The problems on the second sheet. WeBWorK Day03-Lab1 due Monday night.

Classwork

Recall the **Informal Definition**: We say that $\lim_{x \to a} f(x) = L$ if we can make $f(x)$ arbitrarily close to L by taking x sufficiently close to (but not equal to) a.

1. What happens to $f(x) = \dfrac{x^3 - 8}{x - 2}$ as $x \to 2$? What is $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$?

| $x < 2$	$f(x)$	$x > 2$	$f(x)$
1.5	9.25	2.5	15.25
1.9	11.41	2.1	
1.99	11.9401	2.01	
1.999		2.001	12.006001
1.9999		2.0001	12.00060001

As x approaches 2
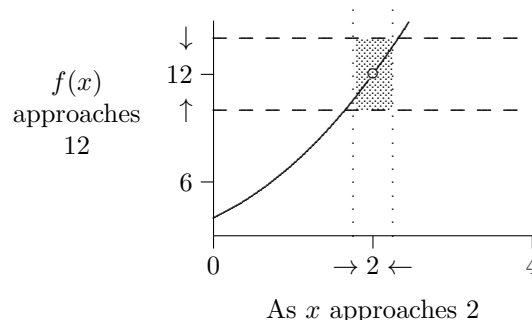
Notice that f is not defined at $x = 2$. What this picture shows is that if **I** choose a small interval around $y = 12$, **YOU** can choose a small interval around $x = 2$ so that the graph of the function over the x-interval (except perhaps right at $x = 2$) lies inside the y-interval near 12. In other words, the graph of f over the selected x-interval (except perhaps right at $x = 2$) lies entirely within the shaded rectangle. Keep this idea in mind in the following examples.

THINK: Is there some algebra that we might do to simplify the process of finding this limit?

2. Determine $\lim_{x \to 1} x^2 + 2x$. What should happen here? Think about the pieces. It $x \to 1$, then $x^2 \to$ _____.
   If $x \to 1$, then $2x \to$ _____. So as $x \to 1$, it follows that $x^2 + 2x \to$ So $\lim_{x \to 1} x^2 + 2x =$ _____.

$x < 1$	$x^2 + 2x$	$x > 1$	$x^2 + 2x$
0.9		1.1	3.41
0.99	2.9601	1.01	
0.999		1.001	3.004001
0.9999	2.99960001	1.0001	

As x approaches 1
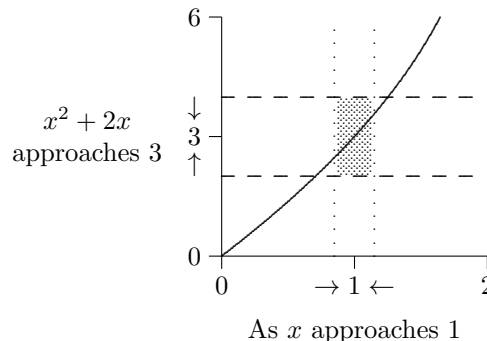
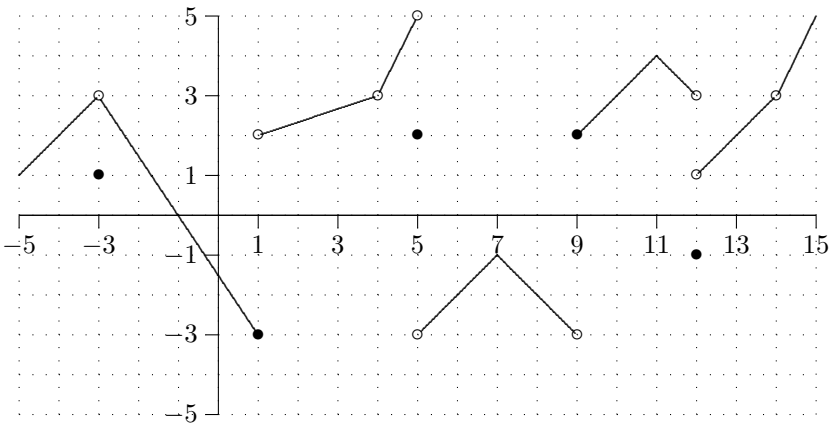**3.** What happens to $\frac{\sin x}{x}$ as $x \to 0$? **This function is not defined at 0**. Does that matter?

| $x < 0$	$\frac{\sin x}{x}$	$x > 0$	$\frac{\sin x}{x}$
-0.5	0.95885108	0.5	
-0.1	0.99833417	0.1	
-0.01		0.01	0.99998333
-0.001		0.001	0.99999983

$\frac{\sin x}{x}$ approaches 1

As x approaches 0
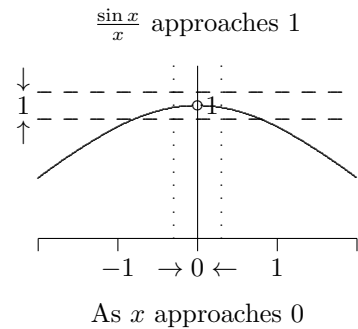
4. Use the graph of f below to determine these limits, if they exist. What do • and ∘ mean?

a	$\lim\limits_{x \to a^-} f(x)$	$\lim\limits_{x \to a^+} f(x)$	$\lim\limits_{x \to a} f(x)$	$f(a)$
-3				
0				
1				
4				
5				
7				
9				
11				
13				
14				

5. Try to determine $\lim\limits_{x \to 0} \sin \frac{\pi}{x}$ by filling in the table of values below. Notice that $\sin \frac{\pi}{x}$ is not defined at 0.

| $x < 0$	$\sin \frac{\pi}{x}$	$x > 0$	$\sin \frac{\pi}{x}$
-0.5	0	0.5	0
-0.1	0	0.1	0
-0.01	0	0.01	0
-0.0011		0.0011	
-0.00021		0.00012	
-0.000013		0.000023	
-0.0000024		0.0000014	
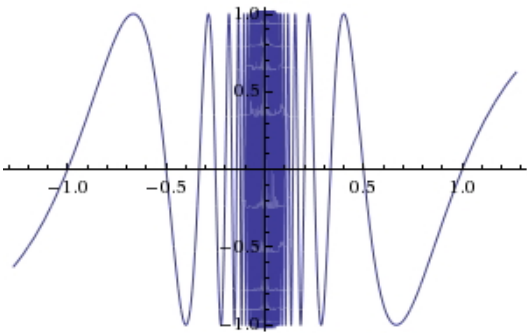
On the right is the graph of $f(x) = \sin \frac{\pi}{x}$. Notice its 'bad behavior' near $x = 0$. Try graphing it on your computer or graphing caccuator. What do you get?

Math 130 Day 3–Hand In Next Time. Name: _____

0. Do WeBWorK Set Day 03-Lab 1. Due Monday night.

1. a) Fill in the table and then graph the function. *Properly indicate any points where the function is not defined.* Make sure your calculator is set in **radians**. Use parentheses with $\tan(2x)$. Use 5 decimal places in your answers.

$x > 0$	$\frac{\tan(2x)}{x}$	$x < 0$	$\frac{\tan(2x)}{x}$
0.5		-0.5	
0.3		-0.3	
0.1		-0.1	
0.01		-0.01	
0.001		-0.001	
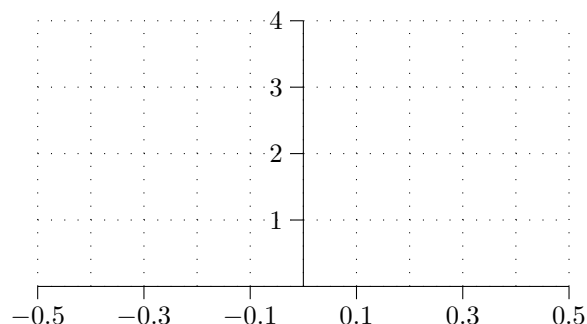
b) Use your table to evaluate these limits if they exist:

$\displaystyle\lim_{x\to 0^-}\frac{\tan(2x)}{x}$	$\displaystyle\lim_{x\to 0^+}\frac{\tan(2x)}{x}$	$\displaystyle\lim_{x\to 0}\frac{\tan(2x)}{x}$

2. a) Use the graph below to determine these limits if they exist.

| a	$\displaystyle\lim_{x\to a^-} f(x)$	$\displaystyle\lim_{x\to a^+} f(x)$	$\displaystyle\lim_{x\to a} f(x)$	$f(a)$
-3				
-2				
2				
4				
5				

b) Draw the rest of a graph of $f(x)$ on the interval from $[7, 11]$ in the figure above so that the graph has **all of these properties**: $f(7) = 3$, $\displaystyle\lim_{x\to 7} f(x) = -2$, $f(9) = 4$, $\displaystyle\lim_{x\to 9^-} f(x) = 0$, and $\displaystyle\lim_{x\to 9^+} f(x) = 4$.

c) True or false: If f is not defined at a, then $\displaystyle\lim_{x\to a} f(x)$ does not exist.

d) Bonus: A function is called **continuous** at $x = a$ if f is defined at a and $\lim_{x\to a} f(x) = f(a)$. At which of the five values of a in the table is f continuous?

3. a) Graph the set of points that satisfies both $|x - 1| < 0.5$ and $|y - 0.5| < 0.25$. (See Problem 2 from Lab 1).
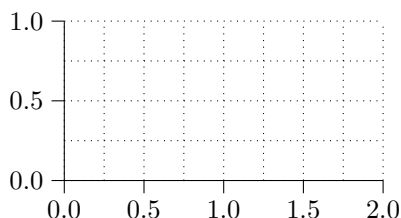
4. Page 68 #36. *Properly indicate any points where the function is not defined.*

$x > 0$	$\frac{|x|}{x}$	$x < 0$	$\frac{|x|}{x}$
2		-2	
1		-2	
0.5		-0.5	
0.1		-0.1	
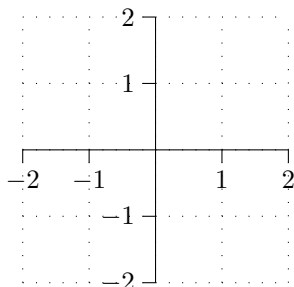
$\lim\limits_{x \to 0^-} f(x)$	$\lim\limits_{x \to 0^+} f(x)$	$\lim\limits_{x \to 0} f(x)$

5. a) [Similar to Lab 1, Problem 7]. An object moves along a straight line so that its position from the origin after t seconds is given by $s(t) = t^2 + 5t$ meters. Find the general formula for the **average velocity** between the points $(1, s(1))$ and $(t, s(t))$. (You can simplify the formula in part (b).)

Average Velocity $=$

b) We saw in class on Friday that the *instantaneous* velocity of the object right at time $t = 1$ is determined by evaluating the limit of the *average* velocity in part (a). Substitute your expression for the average velocity in part (a), determine this limit. (You will need to simplify your formula by factoring. Make sure to use limit notation at each step of the simplification until you arrive at a final answer.)

Instant Vel $= \lim\limits_{t \to 1}$ Average Vel $=$

c) What is the 'tangent slope of the curve' $s(t) = t^2 + 5t$ right at $t = 1$? (No work needed. This is the same as some other number that you have computed.)

6. Calculate and simplify $\frac{f(x+h) - f(x)}{h}$ if $f(x) = x^2 - 2x$. Hint: See Lab 1, problem 8(c). The answers are online for the other parts of this Lab problem. Be very careful not to lose signs in the numerator.