Math 130 Homework: Day 5

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Practice

Review: We will cover the rest of Chapter 2.3 today with all its limit properties. Also see the **online notes** for additional examples. **Read**: Next we will consider infinite limits. Read in Section 2.4.

Important! Most limits that we will consider cannot be calculated simply by plugging in the value into the function. Many limits involve a ratio of the form $\frac{0}{0}$ and, consequently, require **more work** to evaluate them. This work can take the form of factoring, using conjugates, simplifying compound rational expressions, or making use of known limits. *After doing some work, then you should be able to apply the limit properties we have outlined.*

1. a) Try page 76ff #1, 3, 5, 7, 9, 11, 17–29 (odd), 77. These should be easy and quick! A little harder: page 77ff #39, 41, 45, and 43(Hint: write $-x+b$ as $-(x-b)$ and factor!), 47(use conjugates), 49(conjugates again).

b) One-sided limits: #33, 35, and 37.

☞ *Detailed answers to the problems below may be found online at the course website at the end of the Class Notes for Day 5.*

2. Factoring: First check that these are indeterminate of the form $\frac{0}{0}$. If not, are they rational functions and can you apply a limit property. If they are indeterminate, can you factor and then apply a limit law?

a) $\displaystyle\lim_{x\to 4} \frac{x^2-6x+8}{x-4}$ **b)** $\displaystyle\lim_{x\to -2} \frac{x+2}{4-x^2}$ **c)** $\displaystyle\lim_{x\to 5} \frac{x^2-3x-10}{x^2-25}$ **d)** $\displaystyle\lim_{x\to -1} \frac{x^3-x}{x^2-5x-6}$

3. Compound fractions: First check that these are indeterminate of the form $\frac{0}{0}$. Then do more work. Follow the example from class today. Get a common denominator in the numerator. Finally, apply a limit law.

a) $\displaystyle\lim_{x\to 0} \frac{\frac{3}{2x+1}-3}{x}$ **b)** $\displaystyle\lim_{x\to 2} \frac{\frac{1}{x^2}-\frac{1}{4}}{2-x}$ **c)** $\displaystyle\lim_{x\to 1} \frac{\frac{1}{x^2+1}-\frac{1}{2}}{x-1}$

4. Conjugates: First check that these are indeterminate of the form $\frac{0}{0}$. Then do more work. Finally, apply a limit law. You may also need to factor!

a) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$ **b)** $\displaystyle\lim_{x\to 3} \frac{x-3}{\sqrt{x+1}-2}$ **c)** $\displaystyle\lim_{x\to 0} \frac{\sqrt{4-x}-2}{x^2-x}$

Short Hand In for Friday

0. a) WeBWorK set Day04. Due Thursday night.

b) WeBWorK set Day05. Due Sunday night. Start them early!

1. (**This is the same as WeBWorK set Day05, #11. Check your answer there!**) Evaluate the following limits. Show your work. Remember to check whether they are indeterminate.

a) $\displaystyle\lim_{x\to 4} \frac{x^2-6x+8}{x^2-x-12}$ **b)** $\displaystyle\lim_{x\to 3} \frac{\frac{2}{x^2-1}-\frac{1}{4}}{x-3}$ **c)** $\displaystyle\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$ **d)** $\displaystyle\lim_{x\to 1} \frac{x^2-3x+4}{x^2-2}$

2. (**This is the same as WeBWorK set Day05, #12. Check your answer there!**) Let $f(x) = \begin{cases} x^2-x, & \text{if } x \le 3 \\ \frac{x^2+3x}{x-1} & \text{if } x > 3 \end{cases}$.

Determine the following limits if they exist. Show your work.

a) $\displaystyle\lim_{x\to 3^-} f(x)$ **b)** $\displaystyle\lim_{x\to 3^+} f(x)$ **c)** $\displaystyle\lim_{x\to 3} f(x)$

d) Bonus: A function f is **continuous** at a if $\displaystyle\lim_{x\to a} f(x) = f(a)$. Is the function $f(x)$ above continuous at $x = 3$? Why?

Limit Property Review

Limits and Order of Operations Assume that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ and that b is a constant. Then

1. **(Basics).** $\lim_{x\to a} x = a$ and $\lim_{x\to a} b = b$.

2. **(Constant Multiple).** $\lim_{x\to a} bf(x) = b(\lim_{x\to a} f(x)) = bL$.

3. **(Sum or Difference).** $\lim_{x\to a}(f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M$. 'The limit of a sum is the sum of the limits.'

4. **(Product).** $\lim_{x\to a} f(x)g(x) = (\lim_{x\to a} f(x))\cdot(\lim_{x\to a} g(x)) = LM$. 'The limit of a product is the product of the limits.'

5. **(Quotient).** $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$, as long as $M \neq 0$. 'The limit of a quotient is the quotient of the limits.

Assume that m and n are positive integers. Then

6. **(Power).** $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n = L^n$.

7. **(Fractional Power).** Assume that $\frac{n}{m}$ is reduced. Then $\lim_{x\to a}[f(x)]^{n/m} = \left[\lim_{x\to a} f(x)\right]^{n/m} = L^{n/m}$, provided that $f(x) \geq 0$ for x near a if m is even.

8. **(Polynomials).** If $p(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0$ is a degree n polynomial, then $\lim_{x\to a} p(x) = p(a)$. In other words, p is *continuous* at every point.

9. **(Rational Functions).** If $r(x) = \frac{p(x)}{q(x)}$ is a rational function, then for any point a in the domain of $r(x)$ $\lim_{x\to a} r(x) = r(a)$. In other words, r is *continuous* at every point in its domain.

One-sided Limit Properties

Limit properties 1 through 9 above continue to hold for one-sided limits with the following modification for fractional powers Assume that m and n are positive integers and that $\frac{n}{m}$ is reduced. Then

7. a) $\lim_{x\to a^+}[f(x)]^{n/m} = \left[\lim_{x\to a^+} f(x)\right]^{n/m}$, provided that $f(x) \geq 0$ for x near a with $x > a$ when m is even.

b) $\lim_{x\to a^-}[f(x)]^{n/m} = \left[\lim_{x\to a^-} f(x)\right]^{n/m}$, provided that $f(x) \geq 0$ for x near a with $x > a$ when m is even.

Slope of a curve

When trying to calculate the slope of a curve f at a point $(a, f(a))$, we had to use the limit of the secant slopes of the lines through $(a, f(a))$ and $(x, f(x))$.

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a} = \text{Difference Quotient.}$$

To get the slope of the curve right at a we let x approach a and take the limit. (This is why we defined limits!) The resulting slope is m_{tan}, the slope of the tangent line to the curve. It is the line that has the same slope as the curve at $(a, f(a))$.

$$m_{\text{tan}} = \lim_{x\to a} m_{\text{sec}} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}.$$

If $s(t)$ represents position at time t, then $\frac{s(t) - s(a)}{t - a}$ = Ave Vel and Inst Vel = $\lim_{t\to a}$ Ave Vel = $\lim_{t\to a} \frac{s(t) - s(a)}{t - a}$.

☞ In other words, Inst Vel and m_{tan} are really the same thing.