

# Math 130 Homework: Day 6

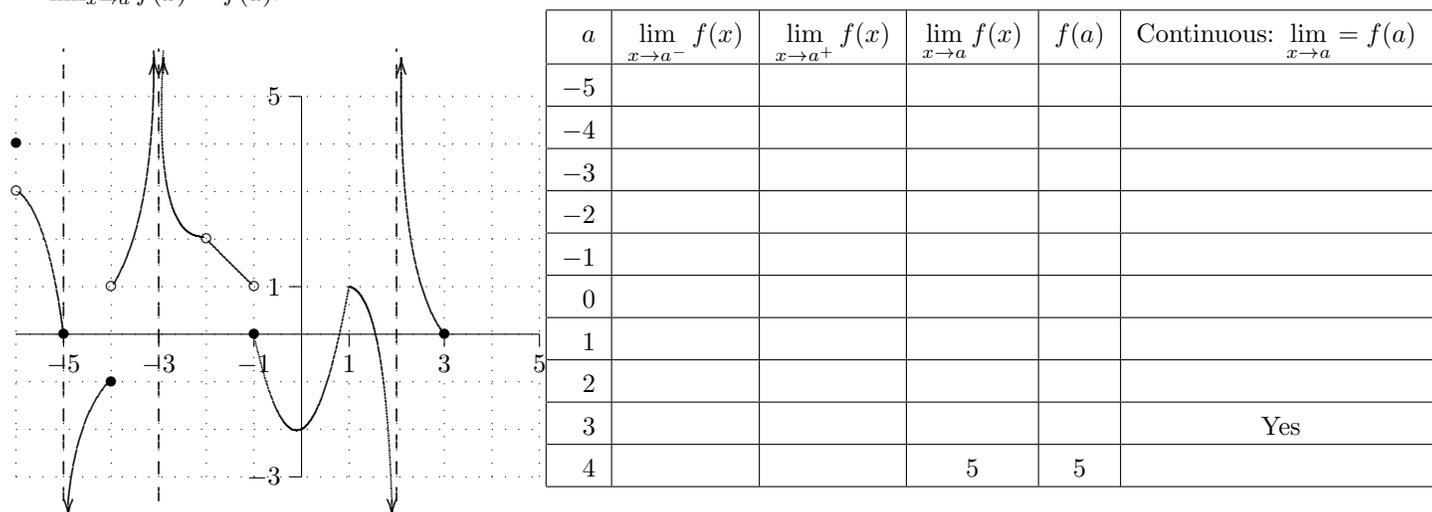
☞ Note the slight change in the Math Intern's Office Hours!

**Office Hours (LN 301/301.5):** M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: <http://math.hws.edu/~mitchell/Math130F16/index.html>.

## Practice

Today we will discuss **infinite limits** and **vertical asymptotes**. This is material in Chapter 2.4.

- Reread Chapter 2.4 carefully. Pay special attention (i.e., know) the definitions on pages 80–82.
- Practice (consult the examples in the text and in online notes): **Infinite limits** Page 84ff #1, 3, 5, 9, 11, 15, 17, 19, 21, 23(good practice), 25, and 41 (good test problem).
- Use the graph of  $f$  to evaluate each of the expressions in the table. Use  $\infty$  or  $-\infty$  if appropriate. For the last few points, complete the graph so that the given information is true. Remember,  $f$  is continuous at  $x = a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$ .



## Hand In

- Make sure to work on WeBWork Set Day 06 due Monday night. Start it early. Remember the Math Intern on Sunday.
- Use pages three and four to complete the written assignment for next class.

**Classwork Examples.** Determine the following:

$$\lim_{x \rightarrow 3^+} \frac{\overbrace{1-x}^{\rightarrow 1-3=-2}}{\underbrace{x-3}_{\rightarrow 0^+}} = \text{_____} \quad \text{and} \quad \lim_{x \rightarrow 3^-} \frac{\overbrace{1-x}^{\rightarrow}}{\underbrace{x-3}_{\rightarrow}} = \text{_____} \quad \text{so} \quad \lim_{x \rightarrow 3} \frac{1-x}{x-3} = \text{_____}$$

**Try:**

$$\lim_{x \rightarrow 2^+} \frac{\overbrace{x}^{\rightarrow}}{\underbrace{(x-5)(x-2)}_{\rightarrow (2-5)0^+}} = \text{_____} \quad \text{and} \quad \lim_{x \rightarrow 2^-} \frac{\overbrace{x}^{\rightarrow}}{\underbrace{(x-5)(x-2)}_{\rightarrow}} = \text{_____} \quad \text{so} \quad \lim_{x \rightarrow 2} \frac{x}{(x-5)(x-2)} = \text{_____}$$

**Try:**

$$\lim_{x \rightarrow 2^+} \frac{\overbrace{x-4}^{\rightarrow}}{\underbrace{x(x-2)^2}_{\rightarrow}} = \text{_____} \quad \text{and} \quad \lim_{x \rightarrow 2^-} \frac{\overbrace{x-4}^{\rightarrow}}{\underbrace{x(x-2)^2}_{\rightarrow}} = \text{_____} \quad \text{so} \quad \lim_{x \rightarrow 2} \frac{x-4}{x(x-2)^2} = \text{_____}$$

## 5-Minute Review: Large Numbers

It will be helpful to remember a couple of simple things about fractions or ratios. Let's use  $0^+$  to indicate a small positive number and  $0^-$  to indicate a small-magnitude negative number. Then

- $\frac{\text{positive number}}{0^+}$  = large positive number
- $\frac{\text{positive number}}{0^-}$  = large - magnitudenegative number
- $\frac{\text{negative number}}{0^+}$  = large - magnitudenegative number
- $\frac{\text{negative number}}{0^-}$  = large positive number

For example,  $\frac{1.2}{0^-}$  represents a large-magnitude negative number, while  $\frac{-1.2}{0^-}$  represents a large positive number. Now let's look at an actual function.

**Example.** Let  $f(x) = \frac{1}{x-1}$ . Examine the behavior of  $f$  near  $x = 1$ .

**Solution.**  $f(x)$  is a rational function and the point  $x = 1$  is not in its domain. However *near*  $x = 1$ , we can make a table of values and plot its graph.

$x < 1$	$f(x) = \frac{1}{x-1}$	$x > 1$	$f(x) = \frac{1}{x-1}$
0.9	-10	1.1	10
0.99	-100	1.01	100
0.999	-1000	1.001	1000
0.9999	-10000	1.0001	10000

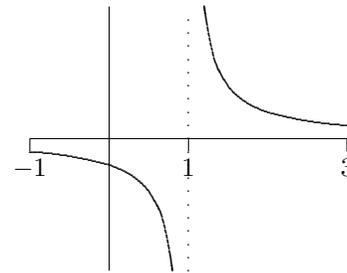


Figure 1: As  $x \rightarrow 1^-$  we see  $f(x) \rightarrow \frac{1}{0^-}$  and as  $x \rightarrow 1^+$  we see  $f(x) \rightarrow \frac{1}{0^+}$ .

You may recognize the graph of  $f(x)$  above as having a vertical asymptote at  $x = 1$ . We will have more to say about that soon. However, when we are interested in the values of a function *near* some point, we should realize that we are talking about limits. In the case of  $f(x) = \frac{1}{x-1}$ , clearly  $\lim_{x \rightarrow 1} \frac{1}{x-1}$  does not exist. However, we can still say *something meaningful and useful* about the behavior of the function near 1.

- Because the values of  $f(x)$  grow *arbitrarily large* or *increase without bound* as  $x \rightarrow 1^+$ , we write  $\lim_{x \rightarrow 1^+} \frac{1}{x-1} = \infty$ .  
The infinity symbol means that  $f(x)$  is getting large. It is not a number and the limit still does not exist in the original sense of the term.
- Likewise we write  $\lim_{x \rightarrow 1^-} \frac{1}{x-1} = -\infty$  because the values of  $f(x)$  *decrease without bound* as  $x \rightarrow 1^-$ .

**Definition: Infinite Limits.** (One-sided) Suppose that  $f$  is defined for all  $x$  near  $a$  with  $x > a$ .

- We write  $\lim_{x \rightarrow a^+} f(x) = \infty$  if  $f(x)$  becomes arbitrarily large for all  $x$  sufficiently close to  $a$  with  $x > a$ .
- Similarly, we write  $\lim_{x \rightarrow a^+} f(x) = -\infty$  if  $f(x)$  is *negative* and becomes arbitrarily large in magnitude for all  $x$  sufficiently close to  $a$  with  $x > a$ .

Now suppose that  $f$  is defined for all  $x$  near  $a$  with  $x < a$ .

- We write  $\lim_{x \rightarrow a^-} f(x) = \infty$  if  $f(x)$  \_\_\_\_\_
- We write  $\lim_{x \rightarrow a^-} f(x) = -\infty$  if  $f(x)$  is \_\_\_\_\_

(Two-sided) Now suppose that  $f$  is defined for all  $x$  near  $a$  (except possibly at  $a$  itself).

- We write  $\lim_{x \rightarrow a} f(x) = \infty$  if both  $\lim_{x \rightarrow a^+} f(x) = \infty$  and  $\lim_{x \rightarrow a^-} f(x) = \infty$
- We write  $\lim_{x \rightarrow a} f(x) = -\infty$  if \_\_\_\_\_

**Math 130 Day 6: Hand in Monday. Name:** \_\_\_\_\_

WeBWork Set Day 06 due Monday night. Do it early. It has both 'regular' and 'infinite' limits.

**1. Warm-up.** Determine the following as we did in class; not all are infinite limits!!

a)  $\lim_{x \rightarrow 3^+} \frac{\overbrace{x^2}^{\rightarrow}}{\underbrace{(x-3)^3}_{\rightarrow}} = \text{_____}$  and  $\lim_{x \rightarrow 3^-} \frac{\overbrace{x^2}^{\rightarrow}}{\underbrace{(x-3)^3}_{\rightarrow}} = \text{_____}$  so  $\lim_{x \rightarrow 3} \frac{x^2}{(x-3)^3} \text{_____}$

b)  $\lim_{x \rightarrow 3^+} \frac{\overbrace{x^2}^{\rightarrow}}{\underbrace{(x-3)^2}_{\rightarrow}} = \text{_____}$  and  $\lim_{x \rightarrow 3^-} \frac{\overbrace{x^2}^{\rightarrow}}{\underbrace{(x-3)^2}_{\rightarrow}} = \text{_____}$  so  $\lim_{x \rightarrow 3} \frac{x^2}{(x-3)^2} \text{_____}$

c) Hint: Factor, then determine what the numerator and denominator approach

$$\lim_{x \rightarrow -2^+} \frac{\overbrace{x-1}^{\rightarrow}}{\underbrace{x^2 - x - 6}_{\rightarrow}}$$

d) Hint: Factor.

$$\lim_{x \rightarrow -2^+} \frac{\overbrace{x^2 + 2x}^{\rightarrow}}{\underbrace{x^2 - x - 6}_{\rightarrow}}$$

**2.** Let  $f(x) = \begin{cases} 2mx^2 + 3, & \text{if } x < -1 \\ 9, & \text{if } x = -1 \\ \frac{m^2 x^2}{2x+3}, & \text{if } x > -1 \end{cases}$ . Determine all values for  $m$  for which  $\lim_{x \rightarrow -1} f(x)$  exists. Show your work which should include limit calculations. (See Lab 2 #5 for a similar problem, with answers online.)

3. Let  $f(x) = 10 + 2x - x^2$ . Evaluate  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ . Use proper limit grammar. (See Lab 2 #8 for a similar problem, with answers online.)

4. No hints this time. Some are infinite, some not.

a)  $\lim_{x \rightarrow 3^+} \frac{-2x + 1}{x - 3}$

b)  $\lim_{x \rightarrow -4^-} \frac{x^2 + 3x - 4}{x + 4}$

c)  $\lim_{x \rightarrow -2^+} \frac{x^2 - 4}{x + 4}$

d)  $\lim_{x \rightarrow 1^+} \frac{x^2}{x^2 - 1}$

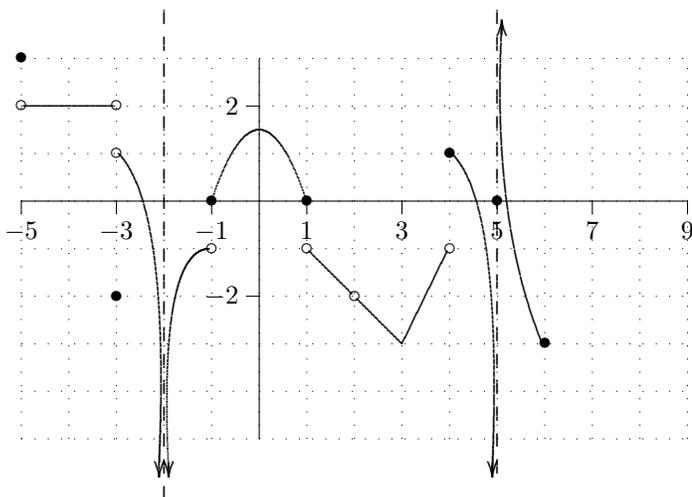
e)  $\lim_{x \rightarrow 2^-} \frac{x^2 + 1}{x(x - 2)}$

5. Beside each of the functions in Problem 4 (all five parts) write “VA” if the function has a vertical asymptote at the point  $x$  is approaching. In one sentence how can you tell from your work if there is a VA?

6. a) Does  $f(x) = \frac{2x - 8}{x^2(x - 4)}$  have a VA at  $x = 0$ ? Explain carefully using limits.

b) Does  $f(x) = \frac{2x - 8}{x^2(x - 4)}$  have a VA at  $x = 4$ ? Explain carefully using limits.

7. a) Use the graph of  $f$  to evaluate each of the expressions in the table or explain why the value does not exist. Note: Use  $+\infty$  and  $-\infty$  when appropriate.



$a$	$\lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow a^+} f(x)$	$\lim_{x \rightarrow a} f(x)$	$f(a)$
-4				
-3				
-2				
-1				
1				
2				
3				
4				
5				

b) Complete the graph of the function above for  $x = 6$  to  $9$  so that all of the following are true:

$$\lim_{x \rightarrow 8^-} f(x) = -2, \quad \lim_{x \rightarrow 8^+} f(x) = +\infty, \quad f(8) = 0.$$