Math 130 Day 07

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Key Concepts

1. The function $f(x)$ has a **vertical asymptote** (VA) at $x = a$ if

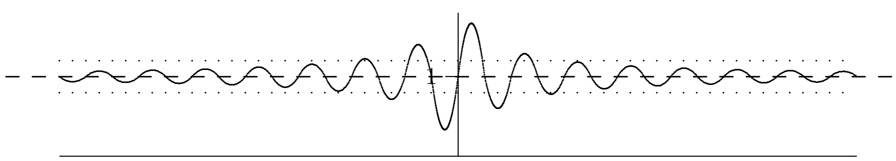
$$\lim_{x \to a^+} f(x) = +\infty \text{ or } -\infty \quad \text{and/or} \quad \lim_{x \to a^-} f(x) = +\infty \text{ or } -\infty.$$

2. The function $f(x)$ has a **removable discontinuity** at $x = a$ if $\lim_{x \to a} f(x)$ exists (and is finite) but does not equal $f(a)$.

3. Limits at Infinity. We say that $\lim_{x \to +\infty} f(x) = L$ if we can make $f(x)$ arbitrarily close to L by taking x sufficiently large. Similarly, $\lim_{x \to -\infty} f(x) = M$ if we can make $f(x)$ arbitrarily close to M by taking x sufficiently large in magnitude but negative.

4. The line $y = L$ is a **horizontal asymptote** (HA) for the graph of $f(x)$ if either $\lim_{x \to +\infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

Note: For each type of asymptote you must compute a particular type of limit.



$\lim_{x \to \infty} f(x) = 1$ (and $\lim_{x \to -\infty} f(x) = 1$) so $y = 1$ is an HA for $y = f(x)$

5. $\boxed{\lim_{x \to +\infty} \frac{1}{x} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{1}{x} = 0.}$ For any positive power r and any constant c, $\boxed{\lim_{x \to +\infty} \frac{c}{x^r} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{c}{x^r} = 0.}$

6. Infinite Limits at Infinity. We say that $\lim_{x \to +\infty} f(x) = \infty$ if $f(x)$ becomes arbitrarily large as x becomes arbitrarily large. We say that $\lim_{x \to \infty} f(x) = -\infty$ if $f(x)$ becomes arbitrarily large in magnitude but negative as x becomes arbitrarily large. Similar definitions are used for $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = -\infty$.

Practice and Classwork

1. Read/Review Chapter 2.5 (pages 88–96) on Limits at Infinity. Most of this material is pretty straightforward if you just think about the size and sign of the numbers involved. There are also additional examples in the online notes. Pay particular attention to Theorems 2.6 and 2.7 in your text.

a) Read the online notes for today's class where there are more examples.

b) Limit practice problems: Page 96 #4, 5, 15–33 odd. These should be very quick to do.

c) Asymptote practice: Page 97 #53 and 57.

2. Determine:

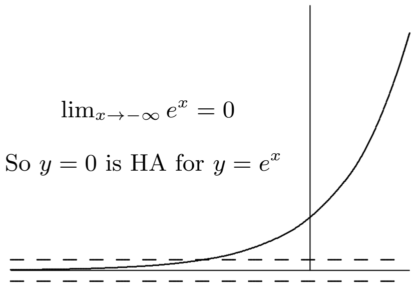
a) $\displaystyle\lim_{x \to -\infty} \frac{x^2 + x - 2}{3x^2 + x}$ **b)** $\displaystyle\lim_{x \to +\infty} \frac{9x + 4}{x^2 - 1}$ **c)** $\displaystyle\lim_{x \to -\infty} \frac{3x^3 + 2x}{8x^2 - x - 6}$ **d)** $\displaystyle\lim_{x \to \infty} \frac{4x^3 + x - 1}{2x^3 + x^2}$

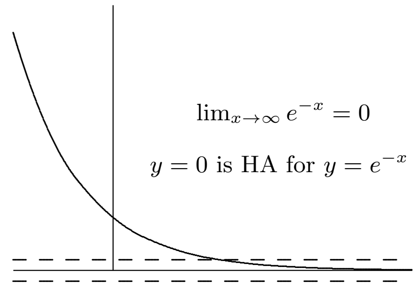
WeBWorK

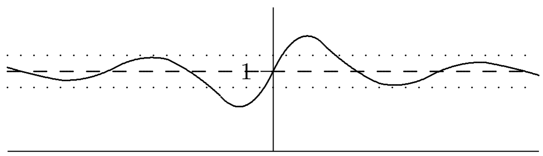
Set Day07F problems are due Thursday. **Note:** For most of these questions you have either only 1, 2, or 3 chances to submit answers. **Be careful!** working the problems out. Remember you can print out the entire problem set from its initial page. Many of the problems are "easy" once you understand the basic ideas. **Use Theorems 2.6 and 2.7** in your text (Chapter 2.5 (pages 85–91)). *You may wish to start them now.* Come for help if you need it or have questions.

Pictures of functions with horizontal asymptotes.

Notice that the function can cross a horizontal asymptote (but not a vertical one). Notice that if $y = L$ is a horizontal asymptote, then it means when x is large enough (in one direction or the either) the function stays within a little horizontal corridor about the line $y = L$...i.e., $f(x)$ gets close to L.

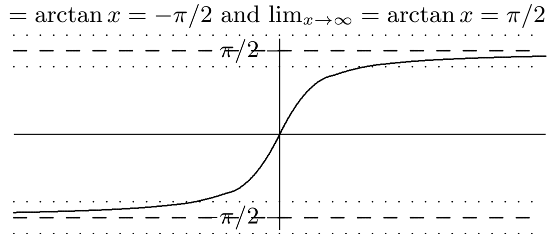






$\lim_{x\to\infty} f(x) = 1$ so $y = 1$ is an HA for $y = f(x)$

$\lim_{x\to-\infty} = \arctan x = -\pi/2$ and $\lim_{x\to\infty} = \arctan x = \pi/2$



$y = \pm\pi/2$ are HA's for $y = \arctan x$

The End Behavior of Polynomials

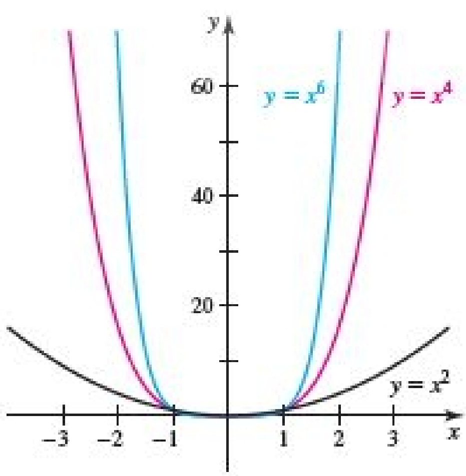
Infinite limits at infinity describe the behavior of all polynomials of degree greater than 0. The simplest examples are provided by functions of the form $f(x) = x^n$ where n is a positive integer. Since positive powers of large numbers are large, this means that for all n,

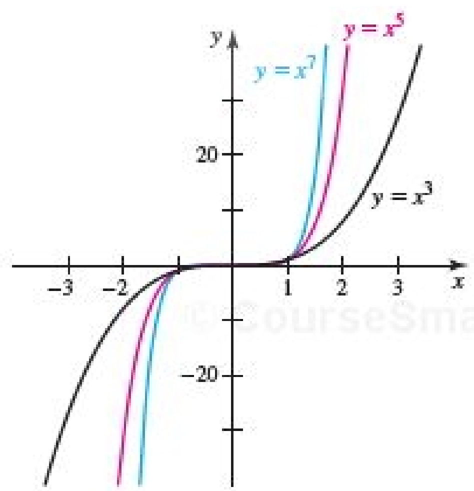
$$\boxed{\lim_{x\to\infty} x^n = +\infty.}$$

Limits at $-\infty$ are only slightly more complicated. Since we are now looking at powers of large magnitude *negative* numbers, the product will be either positive or negative depending on whether n is an *even* or *odd* power. In other words,

$$\boxed{\lim_{x\to-\infty} x^n = \begin{cases} \infty, & \text{if } n \text{ is even} \\ -\infty, & \text{if } n \text{ is odd} \end{cases}}$$

This is illustrated below.





Let's show that when we have any polynomial, its behavior as $x \to \pm\infty$ is completely determined by its highest power. That is if $p(x)$ is a degree n polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, then

$$\boxed{\lim_{x\to\infty} p(x) = \lim_{x\to\infty} a_n x^n \quad \text{and} \quad \lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} a_n x^n.}$$

Math 130, Day 7 Hand-In Wednesday: Name: ________________________

You should be able to determine the limits of each of these functions very easily. Note where you use "highest powers" (HP). Use $+\infty$ and $-\infty$ where appropriate.

1. a) $\lim_{x \to +\infty} -2x^5 + 3x^3 - 1$

b) $\lim_{x \to -\infty} 3x - 2x^2$

c) $\lim_{x \to -\infty} 4x^3 + 19x^2 + 1$

2. a) $\lim_{x \to +\infty} \dfrac{3 - 2x}{3x^3 - 1}$

b) $\lim_{x \to +\infty} \dfrac{3 - 2x}{3x - 1}$

c) $\lim_{x \to -\infty} \dfrac{3 - 2x^2}{3x - 1}$

3. a) $\lim_{x \to +\infty} \dfrac{5x^{3/2}}{4x^2 + 1}$

b) $\lim_{x \to +\infty} \dfrac{5x^{3/2}}{x + 4x^{3/2}}$

4. a) Does $f(x) = \dfrac{6x^2 - 12x}{x^2 - 4}$ have any horizontal asymptotes? Justify your answer with appropriate limit calculations.

b) Where does f have vertical asymptotes? Justify your answer with appropriate limit calculations. Where should you check?

c) Bonus: Does $f(x)$ have a **removable discontinuity**? Justify your answer with appropriate limits. Remember $x = a$ is a removable discontinuity if $\lim_{x \to a} f(x)$ exists BUT is NOT EQUAL to $f(a)$. Use another sheet if needed.