Math 130 Homework: Day 8

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

5-Minute Quiz on Friday

One definition from the list below and one or two limit calculations.

1. The function $f(x)$ has a **vertical asymptote** (VA) at $x = a$ if

$$\lim_{x\to a^+} f(x) = +\infty \text{ or } -\infty \quad \text{and/or} \quad \lim_{x\to a^-} f(x) = +\infty \text{ or } -\infty.$$

2. The line $y = L$ is a **horizontal asymptote** (HA) for the graph of $f(x)$ if either $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

Note: For each type of asymptote you must compute a particular type of limit.

3. A function $f(x)$ is **continuous** at $x = a$ if $\lim_{x\to a} f(x) = f(a)$. Specifically: f is continuous at a if the following three conditions hold:

1. $f(a)$ is defined (i.e., a is in the domain of f).
2. $\lim_{x\to a} f(x)$ exists.
3. $\lim_{x\to a} f(x) = f(a)$.

4. One-sided Continuity.

- $f(x)$ is **continuous from the right at** $x = a$ if $\lim_{x\to a^+} f(x) = f(a)$.
- $f(x)$ is **continuous from the left at** $x = a$ if $\lim_{x\to a^-} f(x) = f(a)$.
- So, if $f(x)$ is both left and right continuous at $x = a$, then it is continuous.

5. A function f has a **removable discontinuity** (RD) at a if the following hold:
1) $\lim_{x\to a} f(x)$ exists (and is finite).
2) $\lim_{x\to a} f(x) \neq f(a)$. Note: $f(a)$ may not even exist.

Hand In at Lab Tomorrow

1. See Page 3. Two quick problems.

2. WeBWorK Set Day08 Due FRIDAY night. Most of these review the basics of continuity and limits.

3. 5-Minute Quiz on Friday. One calculation and one definition.

Practice

Read Chapter 2.6 about Continuity. This section contains several technical points. You must *memorize the definition and checklist on page 99* (or above). We will finish this section next time.

1. Page 108ff, #5, 9, 11. These are very good practice: 13, 15, 17, 19.

2. Read today's online notes for more examples.

3. (Practice from last time) Read/Review Chapter 2.5 (pages 88–96) on Limits at Infinity. Most of this material is pretty straightforward if you just think about the size and sign of the numbers involved. There are also additional examples in the online notes. Pay particular attention to Theorems 2.6 and 2.7 in your text.

a) Limit practice problems: Page 96 #4, 5, 15–33 odd. These should be very quick to do.

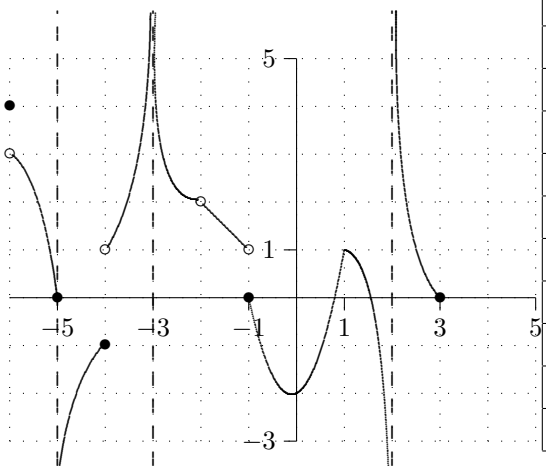
b) Asymptote practice: Page 97 #53 and 57.

Determine:

c) $\lim_{x\to-\infty} \dfrac{x^2+x-2}{3x^2+x}$ **d)** $\lim_{x\to+\infty} \dfrac{9x+4}{x^2-1}$ **e)** $\lim_{x\to-\infty} \dfrac{3x^3+2x}{8x^2-x-6}$ **f)** $\lim_{x\to\infty} \dfrac{4x^3+x-1}{2x^3+x^2}$

Classwork

1. Use the graph of f to evaluate each of the expressions in the table. Use ∞ or $-\infty$ if appropriate. For the last two points in the table, complete the graph so that the given information is true and fill in the rest of the row.



a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Cont? RD? VA?
-5					
-4					
-3					
-2					
-1					
0					
1					
2					
3					Cont
4		-1			RD

2. ☞We will discuss this next problem in class. Fill in the table using the information given. For a few of the numerical answers, there may be more than one correct answer.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Left Cont	Right Cont	Cont	RD
-5	4	2		2				
-4			4	4				
-3	1	2		3				
-2			-3				Cont	
-1			DNE	3		Yes		
0		6			Yes	Yes		
1		3		4	Yes			
2	2	2			No			
3		-1				Yes	No	
4	5	-5			Yes			
5			4				Yes	
6				7			Yes	
7	3						No	Yes
8				1			No	Yes

Lab Ticket: Hand In at Lab Tomorrow. Name: _____

1. This is a good problem to see if you understand the concepts we have been studying. Fill in the table using the information given. For some, several correct answers are possible.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Left Cont	Right Cont	Cont	RD	VA
-1	0	0		DNE					
1		1		4	Yes				
2			2		No				
3		3					No	YES	
4		1				Yes			YES
5			$-\infty$						
6		1					YES		

2. Evaluate these limits. A variety of techniques is required. Use $+\infty$ or $-\infty$, if appropriate.

a) $\lim_{x\to 0^-} \dfrac{x-1}{x^2(x+8)}$

b) $\lim_{x\to 1^+} \dfrac{x-2}{1-\sqrt{x}}$

c) $\lim_{x\to -\infty} \dfrac{3x-2}{\sqrt{4x^2+1}}$

☞ Extra Credit

3. Bonus: Like a Quiz/Test Question.

a) Carefully explain where $f(x) = \dfrac{x^2 + 5x + 6}{x^2 + 2x - 3}$ is NOT continuous. Hint: What type of function is this?

b) Using limits determine where $f(x)$ has (1) vertical asymptotes, and (2) removable discontinuities. [Where should you look.] Use appropriate limits to justify each. See the definitions on p. 1.

c) *Check your understanding.* Give an equation of a rational function with a VA at $x = -2$ and a removable discontinuity at $x = 6$. (Hint: Look back at what happened in the first parts of this problem to create RD's and VA's.)

$f(x) =$