Math 130 Homework: Day 9

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Reading and Practice

Review continuity and one-sided continuity and intervals of continuity (Chapter 2.6). See today's online notes. For next time in Chapter 2.6 read about basic limit properties, the Intermediate Value Theorem and important trig limits.

1. Practice with continuity: Page 108ff #3, 5, 9, 11, 13, 17, 19, 23, 37(good!), 39(good!), and 51(IVT See if you can read and figure this out).

2. Next time: **IVT: The Intermediate Value Theorem**. Assume that f is continuous on the closed interval $[a, b]$ and that k is a number between $f(a)$ and $f(b)$. Then there is at least one number c in (a, b) so that $f(c) = k$.

Classwork

We saw last time:

Theorem 2.10: Polynomials and Rational Functions.

a) A polynomial $p(x)$ is continuous for all x.

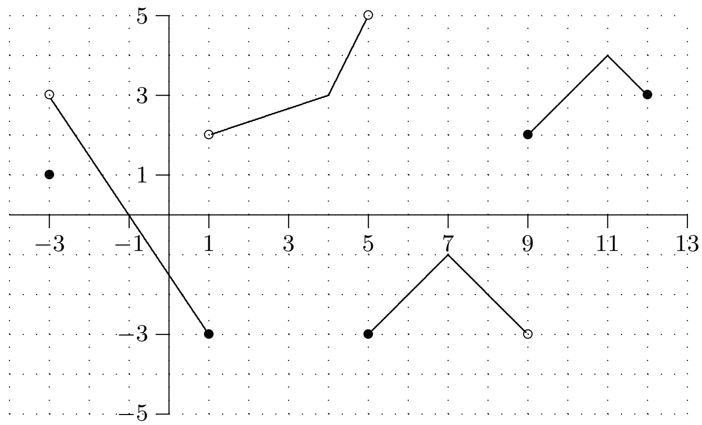
b) A rational function $r(x) = \dfrac{p(x)}{q(x)}$ is continuous for all x for which $q(x) \neq 0$.

1. a) Determine where $r(x) = \dfrac{x^2 + 4x + 4}{x^2 + 2x}$ is continuous. Express your answer as a union of intervals.

b) Determine where $f(x)$ has vertical asymptotes and removable discontinuities. (You must use limits and the definitions.)

Definition: A function $f(x)$ is **continuous on an interval** I if it is continuous at all points of I. If the interval includes one or both endpoints, continuity means continuous from the left or right, as appropriate, as we approach the endpoint from *inside* the interval.

2. Use the graph below to answer these questions. Notice that f is continuous on the interval $(-3, 1]$ since it is continuous from the left at 1 but not from the right at -3. Further f is not continuous on a larger interval containing this one since f is not continuous (from both sides) at either -3 or 1. For each of the following points a, find the largest interval containing a on which f is continuous. Watch the endpoints



a	Interval
0	$(-3, 1]$
3	
5	
7	
9	

3. (Like a HW problem.) Let $f(x) = \begin{cases} x^2 + 4x + 1, & \text{if } x < 1 \\ x^2, & \text{if } x \geq 1 \end{cases}$

a) Show that f is not continuous at 0 using the checklist.

b) Is f left or right continuous at 0?

c) State the intervals of continuity.

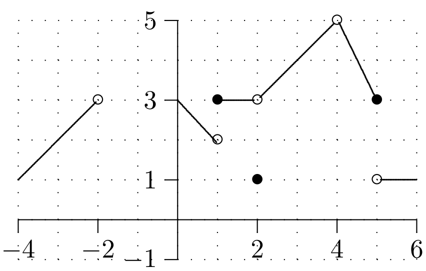
Math 130, Day 9. Hand in Next Class. Name: ________________________

1. a) Page 108 #12. Give the FIRST reason from the continuity checklist that fails for each of the points where $f(x)$ is not continuous.

.

b) Now do page 109 #38. (See the answers to #37 for a similar problem.) Your answer should be a set of intervals starting at 0 and ending at 5.

2. a) Use the graph and the definitions to to fill in the table.



a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Left Cont	Rt Cont	Cont	RD
1								
2								
3								
4								
8	2						Yes	

b) Complete the graph so that f is continuous from the RIGHT at $x = -2$ but NOT from the LEFT.

3. Page 108 #18. Use the continuity checklist.

4. Page 109 #24. Use the theorem the text suggests. (There should be no limit calculations.) Give your answer as a set of intervals.

5. Page 109 #40.

6. Evaluate these limits. Be careful. Use proper limit grammar.

a) $\lim_{x \to 2+} \dfrac{x+1}{x(2-x)^3}$

b) $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 - 2x}$

c) Is the function above continuous at $x = 2$? Or does it have an RD or a VA? Explain using the appropriate definition.

d) $\lim_{x \to 2^-} \dfrac{3x}{\sqrt{x} - \sqrt{2}}$

e) $\lim_{x \to 1} \dfrac{x-1}{\sqrt{2x} - \sqrt{x+1}}$

7. Remember to use the definitions and appropriate limits. Let $f(x) = \begin{cases} x^2 + ax + 1, & \text{if } x < 3 \\ b, & \text{if } x = 3 \\ \dfrac{x^2 - 3x}{x - 3}, & \text{if } x > 3 \end{cases}$

a) Determine a value of b that makes $f(x)$ RIGHT continuous at $x = 3$. Explain.

b) Now that you know the value of b, determine a value of a that makes $f(x)$ LEFT continuous at $x = 3$. Explain.

Math 130 Quiz Day 9. Name: ______________________ **Score: /9**

1. Evaluate these limits. Use ∞ or $-\infty$, if appropriate.

a) $\displaystyle\lim_{x\to 1^+} \frac{x+2}{x(1-x)}$

b) $\displaystyle\lim_{x\to -\infty} \frac{1-4x^4}{8x^3-x^2}$

2. Complete the definition: $f(x)$ is **continuous at** $x = a$