Math 130 Homework: Day 10

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Practice and Reading

1. Review Section 2.6 on Continuity. Next time we will discuss the Formal Definition of Limit in Section 2.7. This is not easy at first. Look at the material and read the Day 11 Classnotes that I will post online.

2. **a)** Practice with continuity: Page 108ff #9, 11, 17, 19, 25, 29, 37(good!), 39, 47, 53, and 55.

b) Review the Intermediate Value Theorem and try page 109 #59 and 63.

3. (Like examples in the online notes. Brief answers at the bottom of the page.) Let $f(x) = \begin{cases} \frac{x^3-4x}{x^2+2x}, & \text{if } x > 0, \\ 2, & \text{if } x = 0 \\ x^3 + 6x + 4, & \text{if } x < 0. \end{cases}$

a) Determine whether f is right continuous, left continuous, or continuous at 0. Show your work with limits.

1) Continuous from the right:

2) Continuous from the left:

3) Continuous (explain using your work above):

b) Determine whether f has a removable discontinuity at 0. Justify with appropriate limits or limit language.

c) Determine whether f has a VA at 0. Justify with appropriate limits or limit language.

d) Use interval notation to describe the intervals of continuity for f.

4. Let $f(x) = \begin{cases} \dfrac{x-1}{\sqrt{x}-1}, & \text{if } x > 1, \\ 2, & \text{if } x = 1 \\ \dfrac{x^2-1}{x-1}, & \text{if } x < 1. \end{cases}$

a) Determine whether f is right continuous, left continuous, or continuous at 1. Show your work with limits.

1) Continuous from the right:

2) Continuous from the left:

3) Continuous (explain using your work above):

b) Determine whether f has a removable discontinuity at 1. Justify with appropriate limits or limit language.

c) Determine whether f has a VA at 1. Justify with appropriate limits that you calculated above.

d) Use interval notation to describe the intervals of continuity for f. Answer: _____

Homework

1. Complete WeBWorK Day 9 and work on WeBWorK Day 10 (due Thursday).

2. Do the problems on pages 3 and 4 of these sheets

3. **Extra Credit, Hand in Next Time**. Do on a separate sheet of paper. Show all work.

Let $f(x) = \begin{cases} \frac{x^2+2x-3}{x-1}, & \text{if } x < -1 \\ mx + b, & \text{if } -1 \le x \le 3 \\ \frac{x^2-8x+15}{x-3}, & \text{if } x > 3 \end{cases}$.

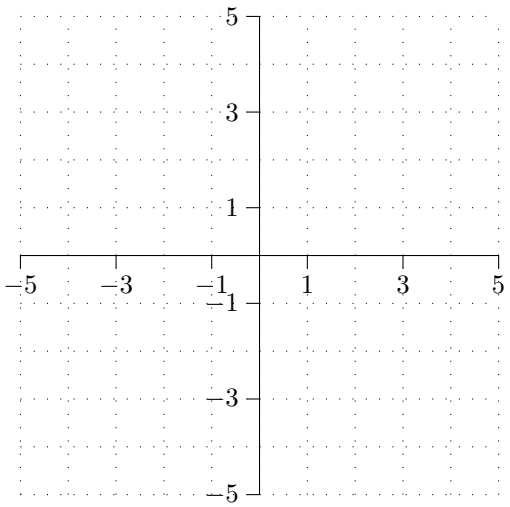
Find the values of the constants m and b so that f is continuous at both $x = -1$ and 3.

Answer to #3: $\lim_{x\to 0^+} f(x) = -2$, $\lim_{x\to 0^-} f(x) = -4$, $f(0) = 2$. Not right or left continuous. Not continuous. No RD because $\lim_{x\to 0} f(x)$ DNE. No VA, neither one-sided limit is infinite.

More Practice

Try and then ask me or Chris, if you have difficulty.

1. Fill in the table below consistent with the information given. For **continuous** use Continuous, Left Cont, Right Cont, RD, VA, as is most appropriate. Then draw a graph which satisfies *all* of the conditions listed in the table.



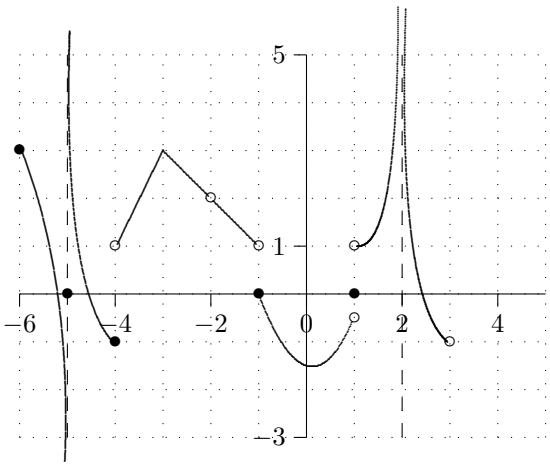
a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	C, LC, RC, RD, VA
-3	1	2		2	
-2			1	DNE	
-1	2	$-\infty$		2	
0			$+\infty$	DNE	
1	2	2			No
2	$-\infty$	$-\infty$			
3	-1				C (L,R)
4	$-\infty$	$+\infty$		0	
5	2	—	—		Left only
-5	—	2	—	2	C on $[-5,-4]$
-4	3	2			C on $[-5,-4]$

2. Fill in the table. You may use $-\infty$ or $+\infty$ in any of the limit columns but NEVER in the $f(a)$ column. Then complete the graph so that all of the following conditions are true.

a) f has a removable discontinuity at $x = 3$ and is defined there.

b) $\lim_{x\to 4^-} f(x) = -\infty$ and $\lim_{x\to 4^+} f(x) = +\infty$

c) $f(5) = 0$ and is left continuous.



a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Cont, RD, VA
-5					
-4					
-3					
-2					
-1					
0					
1					
2					

Math 130, Day 10. Hand In Next Time. Name _____

1. Let $f(x) = \begin{cases} \dfrac{x+1}{(x-4)(x-2)^2}, & \text{if } x < 2 \\ 1/2 & \text{if } x = 2 \\ \dfrac{x^2-2x}{x^2-4}, & \text{if } x > 2 \end{cases}$

a) Determine whether f is left continuous at 2. Show your work with limits. Explain (means use words and the defiinition!)

b) Determine whether f is right continuous at 2. Show your work with limits and explain

c) Determine whether f is continuous at 2. Explain using your work above.

d) Determine whether f has a removable discontinuity at 2. Explain with appropriate limits or limit language.

e) Determine whether f has a VA at 2. Explain with appropriate limits or limit language.

f) Use interval notation to describe the intervals of continuity for f.

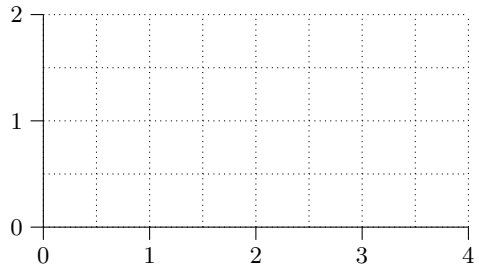
2. (Review your Quiz.) Determine $\lim_{x\to-\infty} \dfrac{1-2x}{\sqrt{36x^2+x-1}}$.

3. Evaluate these limits. Indicate where you used the composition rule for limits and where you used continuity of trig or other functions. See Examples 10.1.21, 10.1.22, and 10.1.26 in the online Notes for Day 10.

a) $\lim_{x\to1} \sin(4x^3-2x^2+x-3)$

b) $\lim_{x\to0} \dfrac{e^x}{6x^2+1+4\cos x}$

4. a) Test Review: Graph the set of points that satisfies both $|x-2|<1$ and $|y-1|<0.5$.



5. Use the Intermediate Value Theorem to show that $f(x) = x^2 \ln x = 5$ at some point on the interval $[1, e]$. See Examples 10.1.28–30 in the online notes. Be sure to explain carefully.