Math 130 Day 19

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Practice and Reading and Classwork

Today we discuss the derivatives of composite functions, that is, functions of the form $y = f(g(x))$. We will see how to write the derivative of such a function in terms of the derivatives of f and g using the **chain rule**. The chain rule makes quick work of differentiating some very complicated looking functions. It's the neatest derivative rule yet!

1. a) Reread and review Chapter 3.7 on the Chain Rule. Do lots of the examples below. This is a derivative rule that you really need to practice! Try to master both versions of the chain rule. Depending on the particular situation or function, one may be easier to use than the other.

b) Version 1: Try page 191 #7, 9, 13, 15, and 17. Version 2: Try page 191 #21, 27, 29, and 33.

c) These are good: Try page 191 #35 (a,b)

d) Additional practice: Page 192 #43. Use the chain rule twice: #45, 49, 53. Test your knowledge: #69 and #37 (See Example 4, page 188).

2. Yvette does a Math 130 Lab 2 times faster than Ulysses. If we express this rate as a a derivative, we would say $\frac{dY}{dU} = 2$. Also suppose that Ulysses works 1.5 times as fast in Lab as Xandy, so we could say $\frac{dU}{dX} = 1.5$. So how many times faster is Yvette than Xandy? From the given information, we should multiply the rates: Yvette is $2 \cdot 1.5 = 3$ times faster than Xandy. In other words:

$$\frac{dY}{dX} = \frac{dY}{dU} \cdot \frac{dU}{dX} = 2 \cdot 1.5 = 3. \tag{1}$$

To measure how Y changes with respect to X we multiply how Y changes with respect to U times how U changes with respect to X. We can do the same with functions.

3. The formula in equation (1) above is called the **chain rule**. It allows us to compute the derivative of a composite function. For example: Suppose $y = f(u) = \sin(u)$ so $\frac{dy}{du} = \cos(u)$. And if $u = g(x) = 3x^2 + 1$, then $\frac{du}{dx} = 6x$. Now their composition is $y = f(g(x)) = \sin(3x^2 + 1)$. To evaluate $\frac{dy}{dx}$ (how y changes with respect to x) we use the chain rule.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos(u) \cdot 6x.$$

Over

The last step is to express the answer completely in terms of x. Since $u = 3x^2 + 1$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos(u) \cdot 6x = 6x\cos(3x^2 + 1).$$

Things to note: In composite functions you must identify the inner function $u = g(x)$ and the outer function $y = f(u)$. Try using the chain rule on the composites in the table.

4. Another way to look at the chain rule: From the table on the other side, we see that $y = f(g(x))$ and that the 'outside derivative' is $\frac{dy}{du} = f'(u)$ and the 'inside derivative is $\frac{du}{dx} = g'(x)$. So by substitution, we can write

$$\underbrace{\frac{d}{dx}\left[f(g(x))\right]}_{\frac{dy}{dx}} = \underbrace{f'(u)}_{\frac{dy}{du}} \cdot \underbrace{g'(x)}_{\frac{du}{dx}}$$

I sometimes like to mix the two versions and say: $\frac{d}{dx}\left[f(g(x))\right] = f'(u) \cdot \frac{du}{dx}$.

Composite: $y = f(g(x))$	Outside: $y = f(u)$	Outside Deriv: $\frac{dy}{du} = f'(u)$	Inside: $u = g(x)$	Inside Deriv: $\frac{du}{dx} = g'(x)$	Chain Rule: $\frac{dy}{du} \cdot \frac{du}{dx}$ $f'(u)\frac{du}{dx} = f'(g(x))g'(x)$
$\sin(3x^2+1)$					
$\cos(e^{2x})$					
$\tan(10x)$					
$(3x^2+1)^5$					
$\sin^5 x = (\sin x)^5$					
$\sqrt{3x^2+1}$					
$\frac{1}{3x^2+1}$					
$\left(\frac{x+5}{3x+1}\right)^4$					
e^{3x^2+1}					

Math 130, Day 19, Due in Lab. Name: ______________________

0. Do WeBWorK Set Day 19. Due Sunday night.

1. Determine these derivatives. Use proper notation. These should take a minute each, once you get the idea.

a) $D_x\left(\cos(3x)\right) =$

b) $D_x\left(\sec(3x^2)\right) =$

c) $D_x(\tan^4 x) =$

d) $D_x\left[\tan(x^4)\right] =$

e) $D_s(e^{2s^4+s}) =$

2. Optional Extra practice: Write out the chain rule derivative for each these, assuming u and v are functions of x.

x) Example $D_x(6\sin(u)) = 6\cos u \cdot \dfrac{du}{dx}$

a) $D_x(7u^6) =$

b) $D_x\left(\sqrt[3]{u^5}\right) =$

c) $D_x\left(e^{3u}\right) =$

d) $D_x\left(\sin(\tan(u))\right) =$

e) $D_x\left(u^3\cos(v))\right) =$