

Math 130 Day 23

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: <http://math.hws.edu/~mitchell/Math130F16/index.html>.

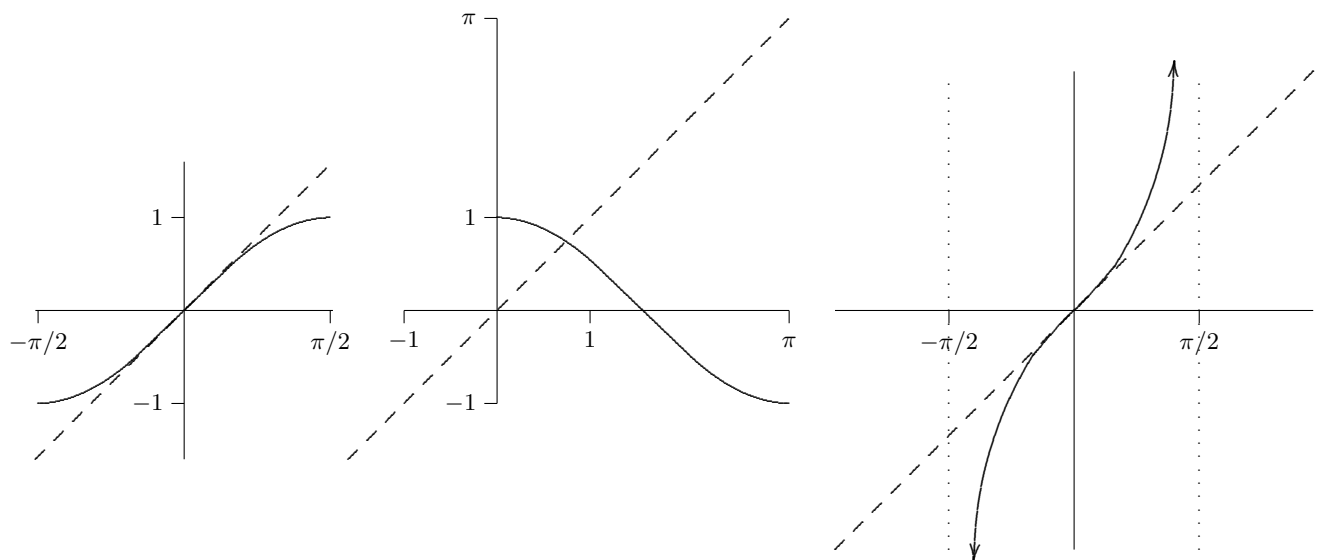
Practice, Reading

We have been discussing inverse functions and today we apply this knowledge to consider inverse trig functions. The main problem is that the standard trig functions do not pass the HLT, so they are not one-to-one, so they don't have inverses. However if we restrict the domains of the trig functions, we can find intervals on which they do pass the HLT. There they do have inverses which are differentiable, and we can find their derivatives.

1. **Reading.** You should have already begun Chapter 3.10 on the Inverse Trig Functions. We will concentrate on the inverse sine and inverse tangent functions. This material is not very familiar since you probably do not have a lot of experience with inverse trig functions. **Please review Chapter 1.4 pages 43 to 47. It will be very worthwhile!** I expect to cover at least the inverse sine and inverse tangent functions today. Note: Your text uses the notation $\sin^{-1} x$ and $\tan^{-1} x$ for these functions but most students find $\arcsin x$ and $\arctan x$ less confusing to use.
 - a) On page 217 review and memorize the derivatives of $\arcsin x$ and $\arctan x$. For the moment you do not need to memorize the other derivatives.
 - b) **I strongly recommend reading and printing today's Class Notes, Day 23. There are several more examples there.**

Class Work: Inverse Trig Functions

The graphs of $\sin x$, $\cos x$, and $\tan x$ are one-to-one (pass HLT) on the intervals shown below. Draw their inverses. (Mark some points on each graph to make this easier.)



2. Fill in the exact values (no decimals) in the table on the left. Then use your first table to fill in the selected values in the second table. Remember, inverses reverse the input and output values of the original functions.

| θ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $-\pi/6$ | $-\pi/4$ | $-\pi/3$ | $-\pi/2$ |
|---------------|---|---------|---------|---------|---------|----------|----------|----------|----------|
| $\sin \theta$ | | | | | | | | | |
| $\cos \theta$ | | | | | | | | | |
| $\tan \theta$ | | | | | | | | | |

| x | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 | $-1/2$ |
|-------------|---|-------|--------------|--------------|---|--------|
| $\arcsin x$ | | | | | | |
| $\arccos x$ | | | | | | |

More Practice

1. Use triangles to help determine

a) $\tan(\arccos(x))$ b) $\cos(\arctan(2x))$ c) $\sin(\arctan 2/3)$ d) $\sec(\arcsin x^2)$

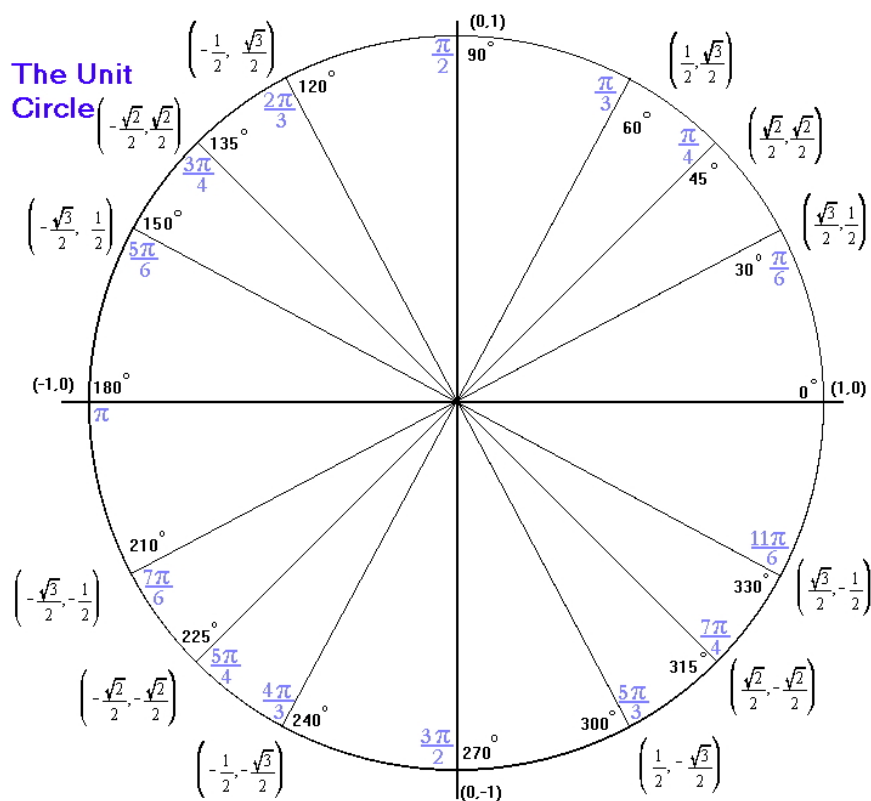
2. Avoid using a calculator to determine each of these values. Use the **chart below**, if necessary.

a) $\sin(\arcsin(1/2))$ b) $\arcsin(\sin(\pi/4))$ c) $\arcsin(\sin(7\pi/4))$ d) $\arccos(\cos(5\pi/3))$

3. Find the derivative of $f(x) = \sin(\arccos x)$ by using an appropriate triangle to rewrite $\sin(\arccos x)$ in a more familiar form. (Don't forget to take the derivative after rewriting.)

4. Now find the derivatives of

a) $\arcsin(e^{3x})$ b) $x^2(\arcsin(2x^3))$ c) $e^{\arctan(4x)}$ d) $\ln|\arctan e^{\sin x}|$



Brief Answers

1. a) $\frac{\sqrt{1-x^2}}{x}$ b) $\frac{1}{\sqrt{4x^2+1}}$ c) $\frac{2}{\sqrt{13}}$ d) $\frac{1}{\sqrt{1-x^4}}$

2. a) $1/2$ b) $\pi/4$ c) $-\pi/4$ d) $\pi/3$

3. Use a triangle to get $\sin(\arccos x) = \sqrt{1-x^2}$. Now $D_x[\sin(\arccos x)] = D_x[\sqrt{1-x^2}] = -\frac{x}{\sqrt{1-x^2}}$.

4. a) $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$ b) $2x(\arcsin(2x^3)) + x^2 \frac{6x^2}{\sqrt{1-4x^6}} = 2x \left[\arcsin(2x^3) + \frac{3x^3}{\sqrt{1-4x^6}} \right]$

c) $-\frac{4e^{\arctan(4x)}}{1+16x^2}$ d) $-\frac{1}{\arctan e^{\sin x}} \cdot \frac{e^{\sin x} \cos x}{1+e^{2 \sin x}}$

Day 23 Hand In Next Class. Name: _____

0. WeBWork Day 23 due **Tuesday**. Practice Problems for Exam 2 are now Online. Covers material through today's class.

1. Fill in the exact values (no decimals) in the table on the left. Then use your first table to fill in the selected values in the second table. Remember, inverses reverse the input and output values of the original functions.

| θ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---------------|---|---------|---------|---------|---------|
| $\sin \theta$ | | | | | |
| $\cos \theta$ | | | | | |

| x | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 |
|-------------|---|-------|--------------|--------------|---|
| $\arcsin x$ | | | | | |
| $\arccos x$ | | | | | |

2. These are NOT derivative questions. (See class and online notes.) **Show your work using triangles** to help determine simpler expressions for

a) $\tan(\arcsin 4/7)$ [WeBWork Set Day 23, #4]

b) $\sin(\arccos(4/9))$ [WeBWork Set Day 23, #4]

c) $\sin(\arccos(x))$ [WeBWork Set Day 23, #7]

d) $\cos(\arctan(2x))$ [WeBWork Set Day 23, #7]

3. Determine the derivatives of the following inverse trig functions. Use correct 'grammar.'

a) $y = \arcsin(4x^2)$

b) $y = \sin(2x)\arcsin(2x)$ [WeBWorK Set Day 23, #10]

c) $y = \arcsin(e^{7x})$ [Similar to WeBWorK Set Day 23, #8]

d) $y = \arctan(6x^2)$ [Similar to WeBWorK Set Day 23, #9]

e) $y = \arctan(\ln x)$ [WeBWorK Set Day 23, #14]

4. Determine the derivatives of the following functions. Use correct 'grammar.'

a) $y = 8^{\arctan(x^2)}$ [WeBWorK Set Day 23, #15]

b) $y = (\arcsin x)^{\cos x}$. What technique must you use? [See the back of the page for the Quiz problem from lab. This is easier.]

Math 130, Lab 8 Quiz. Names: Answer

1. Determine $D_x [(6x^2 + 1)^{4^{\cos x}}]$. use logarithmic differentiation

$$y = (6x^2 + 1)^{4^{\cos x}}$$

$$\ln y = \ln (6x^2 + 1)^{4^{\cos x}} = 4^{\cos x} \ln(6x^2 + 1)$$

$$\frac{1}{y} \frac{dy}{dx} = 4^{\cos x} \ln 4 \cdot (-\sin x) \ln(6x^2 + 1) + 4^{\cos x} \cdot \frac{12x}{6x^2 + 1}$$

$$\frac{dy}{dx} = y \cdot \left[4^{\cos x} (-\ln 4 \cdot \sin x \cdot \ln(6x^2 + 1) + \frac{12x}{6x^2 + 1}) \right]$$

$$\frac{dy}{dx} = (6x^2 + 1)^{4^{\cos x}} \left[4^{\cos x} (-\ln 4 \cdot \sin x \cdot \ln(6x^2 + 1) + \frac{12x}{6x^2 + 1}) \right]$$

Math 130, Lab 8 Quiz. Names: _____

1. Determine $D_x [(6x^2 + 1)^{4^{\cos x}}]$