## Math 130 Day 24

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. Math Intern: Sun through Thurs: 3:00-6:00, 7:00-10:00pm. Website: Use the links at the course homepage on Canvas or go to my course Webpage: http://math.hws.edu/~mitchell/Math130F16/index.html.

## Exam II Coverage

The test covers the material since the last exam. Chapter $3.1-3.5$ and $3.6-3.10$. This includes the topics below. Caution: This is at least $90-95 \%$ of what we have covered, but there may be other topics I have forgotten to list. Review labs and previous homework. All the answers are online. Try some of the Practice Problems online. at our website.

1. $m_{\mathrm{sec}}=\frac{f(x)-f(a)}{x-a}, m_{\mathrm{tan}}=\lim _{x \rightarrow a} m_{\mathrm{sec}}=\lim _{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$.
2. Another form: $m_{\mathrm{sec}}=\frac{f(a+h)-f(a)}{h}$ and $m_{\tan }=f^{\prime}(a)=\lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$.
3. The definition of the derivative: $f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$, if the limit exists. Computing derivatives of functions using this definition.
4. $f^{\prime}(x)$ is the slope function for $f(x)$. Finding tangent lines.
5. $f^{\prime}(x)$ is a rate of change (velocity, acceleration, flow rate).
6. Sketch the graph of $f^{\prime}$ from the graph of $f$ (see p. 136).
7. Differentiable implies Continuous (so NOT Continuous implies NOT Differentiable). But: Can you draw examples of continuous functions that are not differentiable.
8. Derivative rules: Constant Multiple, Sum, Product, Quotient, Power, and Chain Rules.
9. Derivative formulas for: $e^{x}, \ln x, \ln |x|, \sin x, \cos x, \tan x, \sec x, \arcsin x, \arctan x$, and $b^{x}$ (where $b>0$ is a constant). Know the the chain rule versions of these derivative rules.
10. The definition of the number $e$ (see page 148). Two important trig limits: $\lim _{x \rightarrow 0} \frac{\sin x}{x}=1$ and $\lim _{x \rightarrow 0} \frac{\cos x-1}{x}=0$
11. Implicit differentiation and finding tangent lines, again.
12. Logarithmic differentiation for functions of the form $y=[f(x)]^{g(x)}$.

## Practice

Today we will finish with the inverse trig functions and work on Related Rates problems. These are interesting applications of implicit differentiation. Exam II in lab on Thursday. See Practice Problems online.

1. a) Read Chapter 3.11 and today's online notes about related rates.
b) Try problems page $227 \mathrm{ff} \# 5,9,12\left(\mathrm{ans}-.5 /\left(3 \times 12^{2}\right) \approx-0.00116 \mathrm{ft} / \mathrm{s}, 13,15\right.$ (the question is saying that $d V / d t=$ $k \cdot 4 \pi r^{2}$ for some constant $k$ ), 23, 27 (you want $d V / d t$ ), 33 .
2. Assume that $x, y, \theta$, and $z$ are functions of $t$. Find the derivatives with respect to $t$ of each of these relations. (Answers on the back.)
a) $x^{2}+y^{2}=9$
b) $\tan \theta=\frac{y}{30}$
c) $z=x y$
d) $z^{2}=x^{2}+y^{2}$
e) $z=\pi x^{2} y$

## Class Work: Related Rates

These are all done in detail in the online notes. (The example numbers refer to the online notes.)

1. Example 1. Two students finish a conversation and walk away from each other in perpendicular directions. If one person walks at $4 \mathrm{ft} / \mathrm{sec}$ and the other at $3 \mathrm{ft} / \mathrm{sec}$, how fast is the distance between the two changing at time $t=10 \mathrm{sec}$ ?
2. Example 6. An ice block (draw and label a diagram) with a square base is melting in the sun at a steady rate of $48 \mathrm{~cm}^{3} / \mathrm{hr}$. If the height is decreasing at a rate of $0.5 \mathrm{~cm} / \mathrm{hr}$, how fast is the surface area of the block changing at the same moment?
3. Example 3. A kite 100 feet above the ground moves horizontally at a rate of $8 \mathrm{ft} / \mathrm{s}$. At what rate is the angle between the string and the vertical direction changing when 200 ft of string have been let out? Hint: Let $x$ denote the horizontal side opposite $\theta$. Use trig to relate $\theta$ and $x$.

4. Answers for Practice Problem 2
a) $2 x \frac{d x}{d t}+2 y \frac{d y}{d t}=0$
b) $\sec ^{2} \theta \frac{d \theta}{d t}=\frac{1}{30} \frac{d y}{d t}$
c) $\frac{d z}{d t}=y \frac{d x}{d t}+x \frac{d y}{d t}$
d) $2 z \frac{d z}{d t}=2 x \frac{d x}{d t}+2 y \frac{d y}{d t}$
e) $\frac{d z}{d t}=2 \pi x y \frac{d x}{d t}+\pi x^{2} \frac{d y}{d t}$
5. Answers for Practice Problem 0 on the next page.
a) $V=x^{2} y \Longrightarrow \frac{d V}{d t}=2 x y \frac{d x}{d t}+x^{2} \frac{d y}{d t}$
b) $S=2 \pi r^{2}+2 \pi r h \Longrightarrow \frac{d S}{d t}=4 \pi r \frac{d r}{d t}+2 \pi h \frac{d r}{d t}+2 \pi r \frac{d h}{d t}$
c) $\theta=\arctan \frac{x}{4} \Longrightarrow \frac{d \theta}{d t}=\frac{1}{1+\frac{x^{2}}{16}} \cdot \frac{1}{4} \frac{d x}{d t}=\frac{16}{16+x^{2}} \cdot \frac{1}{4} \frac{d x}{d t}=\frac{4}{16+x^{2}} \frac{d x}{d t}$

Math 130 Day 24, Hand In. Name:
0. Work on WeBWork set Day 24 due Thursday night. This reviews inverse trig and logarithmic differentiation and is good practice for the exam on this recent material. Now try this practice problem before working on the actual problems to hand in. Assume that $x, y, \theta, r, V, S$, and $h$ are functions of $t$. Find the derivatives with respect to $t$ of each of these relations. (Check: Answers on the previous page.)
a) $V=x^{2} y$
b) $S=2 \pi r^{2}+2 \pi r h$
c) $\theta=\arctan \left(\frac{x}{4}\right)$

1. Determine the derivatives of the following functions.
a) (WeBWork Set Day $24, \# 3$.) $y=x^{3} \arcsin (4 x)$
b) (WeBWork Set Day 24, \#4.) $y=\arctan \left(\sqrt{4 x^{2}-1}\right)$
2. Page $227 \# 10$. Fill in the outline used in class.
1) Given (known) rate(s):
2) Unknown rate:
3) Relation:
4) Rate-ify (take derivative):
5) Substitute known values to determine the unknown rate.
3. An animated rectangle in a Baby Einstein video is changing. Find the rate of change in its area $A$ if $d h / d t=-2 \mathrm{~cm} / \mathrm{s}$ and $d \ell / d t=3 \mathrm{~cm} / \mathrm{s}$ at the instant when $h=12 \mathrm{~cm}$ and $\ell=5 \mathrm{~cm}$. Use the method in problem 2.
4. The volume of a cone is $V=\frac{1}{3} \pi r^{2} h$. Find the rate of change in the volume if $d r / d t=2 \mathrm{in} / \mathrm{min}$ and $d h / d t=6 \mathrm{in} / \mathrm{min}$ at the time when $r=6$ in and $h=18 \mathrm{in}$. Use the method in problem 2.
