Math 130 Day 27

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Today we will examine a consequence of the Critical Point Theorem and the Extreme Value Theorem. If there is time we will begin to discuss the Mean Value Theorem. This is the most important theorem we will encounter this term.

Major Theorems in Differential Calculus

These theorems are critical to most of what we will do for the rest of the term. **Memorize #1–3** today. Eventually you should know all of them.

1. **EVT: Extreme Value Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Then f has both an absolute maximum value and an absolute minimum value on the interval $[a, b]$.

2. **CPT (CNT): Critical Point (Number) Theorem.** If f has a local extremum at c, then c is a critical point of f.

3. **CIT: The Closed Interval Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

Definitions

1. Let f be a function defined on an interval I containing the point c.
   - **a)** f has an **absolute (global) maximum** at c if $f(c) \geq f(x)$ for all x in D. The number $f(c)$ is the **maximum value** of f.
   - **b)** f has an **absolute (global) minimum** at c if $f(c) \leq f(x)$ for all x in D. The number $f(c)$ is the **minimum value** of f.
   - **c)** If $f(c) \geq f(x)$ for all x in some open interval containing c, then $f(c)$ is a **relative (local) maximum** value of f. (Or: f has a local max at c.)
   - **d)** If $f(c) \leq f(x)$ for all x in some open interval containing c, then f is a **relative (local) minimum** value of f. (Or: f has a local min at c.)

Practice

1. **a)** Review Chapter 4.1. Read Chapter 4.6 through page 293. Also read today's online notes. Next week we will spend most of the time working in Chapter 4.2. So read ahead. There is a lot of "good" stuff in this section. It is the payoff for all the hard work this term.
   - **b)** Practice: Page 242ff #13–19 (odd), 25, 29, 31, 39, 43 (good practice for one of the hand in problems), 55, 59, 61, and 65.

Next time:

MVT: The Mean Value Theorem. Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there is some point c between a and b so that

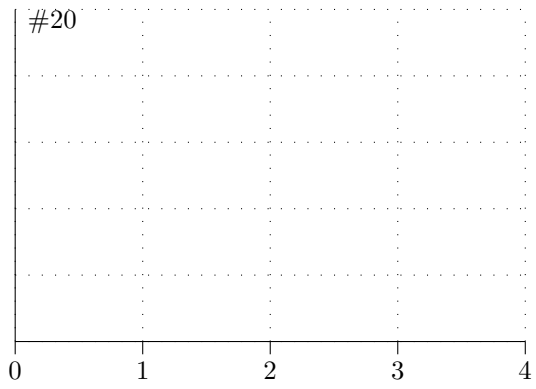
$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{which means} \qquad f(b) - f(a) = f'(c)(b - a).$$

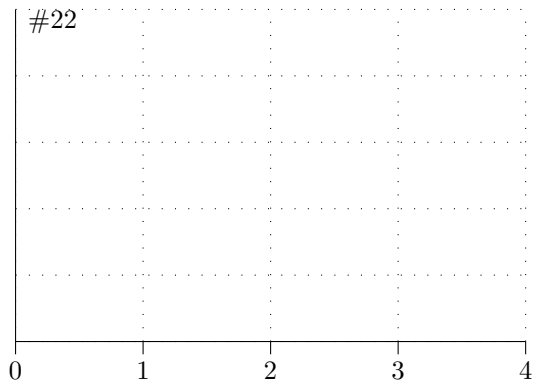
[You should be able to draw a graph that illustrates this.]

Math 130, Day 27. Hand In Wednesday: Name: _____

0. There are two WeBWorK sets Day 26 which reviews critical numbers (due Wednesday) and Day 27 which reviews extrema (due Thursday). Both will help you with this homework assignment. Try some first.

1. Page 242–2433 #20 and 22. (Read the instructions on page 242.)





2. a) (Basics.) Find the critical number(s) of the function $f(x) = -2x^2 + 8x$. [WeBWorK Day 27, #1.]

b) Use this information and the CIT to determine the extreme values (absolute max and absolute min) of $f(x)$ on the interval $[-1, 3]$.

c) Determine the extreme values $f(x)$ on the interval $[-3, 0]$.

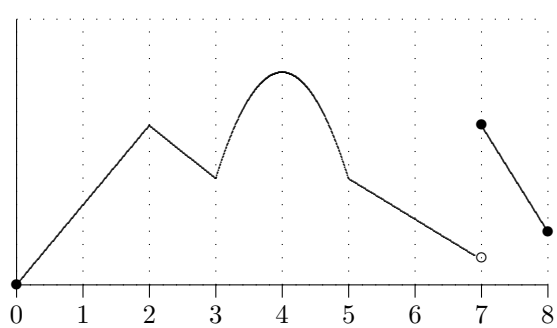
3. Use the same process as in the previous problem to determine the extreme values of $f(r) = \dfrac{3r}{r^2+1}$ on the interval $[-4, 0]$. [WeBWorK Day 27, #7.]

4. Page 243 #44. (Use the same process.) [WeBWorK Day 27, #5.]

5. Use the same process to determine the extreme values of $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$.

6. f is defined on $[0, 8]$ as in the graph below. List x coordinates of all the points which satisfy the given property.

Property	List solutions in order
absolute max	
relative max	
relative max but not critical #	
not differentiable but relative max	
absolute min	
absolute min but not relative min	
critical number	
critical # but not a relative extrema	
critical # and $f'(x)$ DNE	
$f'(x) = 0$ but not critical	



The answers (in mirror writing) are given on the bottom of the next page. Grade yourself. Give yourself 1/2 point for each question you got entirely correct. Score: /5

7. [WeBWorK Day 27, #8.] If $y = x^{2x}$ on $[0.1, 1]$, find the absolute extreme values and the points at which they occur. Be careful. You will need a calculator to compare the values.

Property	List all points in numerical Order
absolute max	4
relative max	2, 4, 7
relative max but not critical #	Impossible by CPT
not differentiable but relative max	2, 7
absolute min	0
absolute min but not relative min	0
critical number	2,3,4,5,7
critical # but not a relative extrema	5
critical # and $f'(x)$ DNE	2,3,5,7
$f'(x) = 0$ but not critical	Impossible by Definition of CP