Math 130 Day 28

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Key Definitions

1. **Definition.** Let f be a function defined on an interval I containing the point c.
   - **a)** f has an **absolute (global) maximum** at c if $f(c) \geq f(x)$ for all x in D. The number $f(c)$ is the **maximum value** of f.
   - **b)** f has an **absolute (global) minimum** at c if $f(c) \leq f(x)$ for all x in D. The number $f(c)$ is the **minimum value** of f.
   - **c)** If $f(c) \geq f(x)$ for all x in some open interval containing c, then $f(c)$ is a **relative (local) maximum** value of f. (Or: f has a local max at c.)
   - **d)** If $f(c) \leq f(x)$ for all x in some open interval containing c, then f is a **relative (local) minimum** value of f. (Or: f has a local min at c.)

Key Continuous Function Theorems

1. **IVT: Intermediate Value Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Let L be any number between $f(a)$ and $f(b)$. Then there is some point c in (a, b) so that $f(c) = L$.

2. **EVT: Extreme Value Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Then f has both an absolute maximum value and an absolute minimum value on the interval $[a, b]$.

Major Theorems in Differential Calculus

These theorems are critical to most of what we will do for the rest of the term. **Memorize #1–3** today. Eventually you should know all of them.

1. **CPT: Critical Point Theorem.** If f has a local extremum at c, then c is a critical point of f.

2. **CIT: The Closed Interval Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

3. **MVT: The Mean Value Theorem.** Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there is some point c between a and b so that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{which means} \qquad f(b) - f(a) = f'(c)(b - a).$$

   [You should be able to draw a graph that illustrates this.]

4. **The Increasing/Decreasing Test.**
   - **a)** If $f'(x) > 0$ on an interval, then f is increasing there.
   - **b)** If $f'(x) < 0$ on an interval, then f is decreasing there.

Reading and Practice

Today we will begin a discussion about the Mean Value Theorem. This is the most important theorem his term.

1. a) Reread Chapter 4.6 up through page 293. Today's class should make this seem straightforward.

b) Next we will be considering consequences of the the MVT.

c) **Begin Chapter 4.2** which has lots of important results. Derivatives can tell us whether a curve is increasing or decreasing and whether it is bending up or down. This is useful for graph sketching.

2. a) Practice: Review locating extreme values on closed intervals. Most of these were assigned previously: Page 242ff #29, 31, 39, 43 (good practice for one of the hand in problems), 55, 59, 61, and 65.

b) Practice with the MVT: Page 295 #17, 19, 23 (be careful!), and 29.

Hand In at Lab

0. Finish WeBWorK sets Day 27 (Thursday) and begin Day 28 (Sunday).

1. Hand in the attached sheet at lab. These should be quick.

Math 130: Hand in at Lab 10. Name: ________________________

1. Designer Functions. Draw a function that satisfies the given conditions or **explain why this is impossible**. Make sure that your function is defined (has an output value) for every x in the given interval. [Each part is a separate problem.]

a) A continuous function on $[0, 10]$ which has an absolute min at $x = 2$ and has relative but not absolute max at $x = 6$.

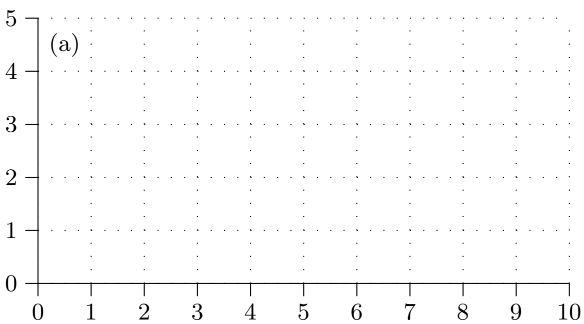
b) A function on $[0, 6]$ which has no absolute max.

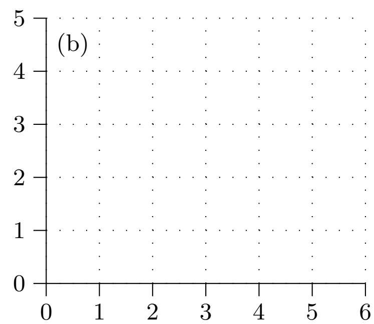
c) A continuous function on $(0, 6)$ which has no absolute max.

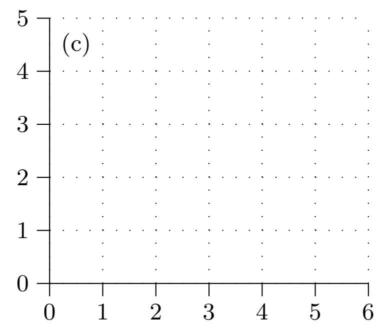
d) A function on $[0, 6]$ for which $f(0) = 3$ and $f(6) = -2$ and which is never 0.

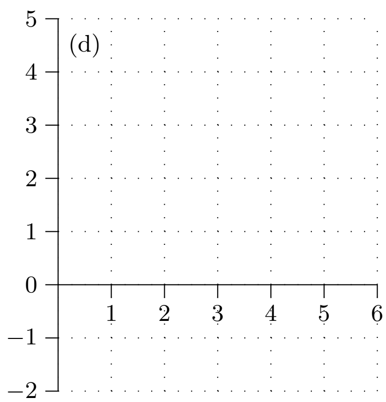
e) A differentiable function on $[0, 6]$ which has no absolute max. (Think: If f is differentiable, what else can you say about it?)

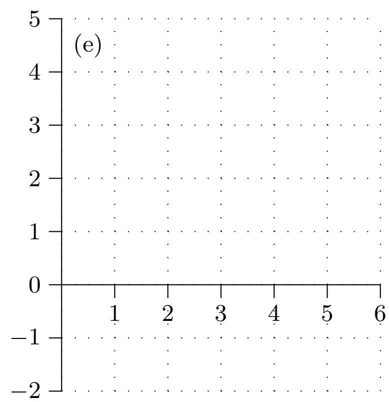
f) A function which illustrates the Mean Value Theorem. (Mark the tangent and secant lines.)

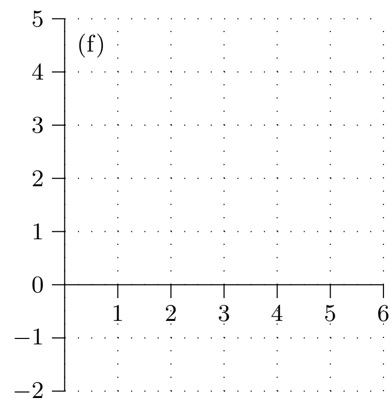






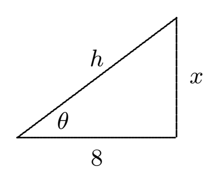






2. Suppose that one leg of a right triangle has a fixed length of 8 cm. Let x denote the other leg of the triangle. Assume that $dx/dt = 2\text{cm/sec}$. See figure below.

a) If h represents the length of the hypotenuse, find dh/dt when $x = 6$ cm.



The problem continues on this side. Use your work from the other side, if it helps

b) If θ is the angle shown (other side), find $d\theta/dt$ when $x = 6$ cm.

c) If A represents the area of the triangle, find dA/dt when $x = 6$ cm. Be careful. h is the hypotenuse, not the height.

3. a) Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$. Determine the absolute extreme points on $[-1, 2]$. WeBWorK Day28 #1.

b) State the name of the theorem that you used: