# Math 130 Day 29

**Office Hours (LN 301/301.5):** M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Today we will continue our discussion of critical points with two important results related to the first derivative: The Increasing/Decreasing Test and the First Derivative Test.

**1. CPT: Critical Point Theorem.** If $f$ has a local extremum at $c$, then $c$ is a critical point of $f$.

**2. CIT: The Closed Interval Theorem.** Let $f$ be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of $f$ occur either at critical points of $f$ on the open interval $(a, b)$ or at the endpoints $a$ and/or $b$.

**3. MVT: The Mean Value Theorem.** Let $f$ be continuous on a closed interval $[a, b]$ and differentiable on $(a, b)$. Then there is some point $c$ between $a$ and $b$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

**4. The Increasing/Decreasing Test.**

**a)** If $f'(x) > 0$ on an interval, then $f$ is increasing there.

**b)** If $f'(x) < 0$ on an interval, then $f$ is decreasing there.

**5. The First Derivative Test.** Let $c$ be a critical number of a continuous function.

**a)** If $f'$ changes from positive to negative at $c$, then $f$ has a local maximum at $c$.

**b)** If $f'$ changes from negative to positive at $c$, then $f$ has a local minimum at $c$.

**c)** If $f'$ does not change sign at $c$, then $f$ does **not** have a local extremum at $c$.

## Practice/Reading

**1. a)** **Important Practice.** Read all of Section 4.2. Next time we will concentrate on the role of the second derivative in determining concavity (the bend of a curve). Look ahead to Section 4.3.

**b)** Try these for improving basic skills: Page 256 #11, 13, 17, 27, 33.

**c)** These questions asks you to put it all together: Page 257 #39(basic), 41, 43, and 47. Be sure to simplify the derivatives.

## Math 130, Day 29. Hand In. Name: ________________________

See the Day 29 and Day 28 online notes at our website (address at top of page) for more examples. WeBWorK set Day 29 due Tuesday night. Complete the Pre-Lab assignment below. Use pencil. **Neatness and accuracy count! These should be perfect. Messy or late papers will be rejected.**

**1. a)** Look in your text and write out the definition of when $c$ is a **critical point** of $f$.

**b)** Explain from the definition why $c = 3$ is NOT a critical point of $f(x) = \frac{x}{x-3}$.

**2.** Determine the critical points of each of these functions. Determine the intervals where each function is increasing or decreasing. Classify each critical point as relative max, relative min, or neither. Use the number line to organize your information.

**a)** $f(x) = x^3 e^x$ (See Lab 10, #4.)

Number Line for $f'$ ________________________________

**b)** $f(x) = 3(x^2 - 1)^{5/3} + 2$ (See Lab 10, #6.)

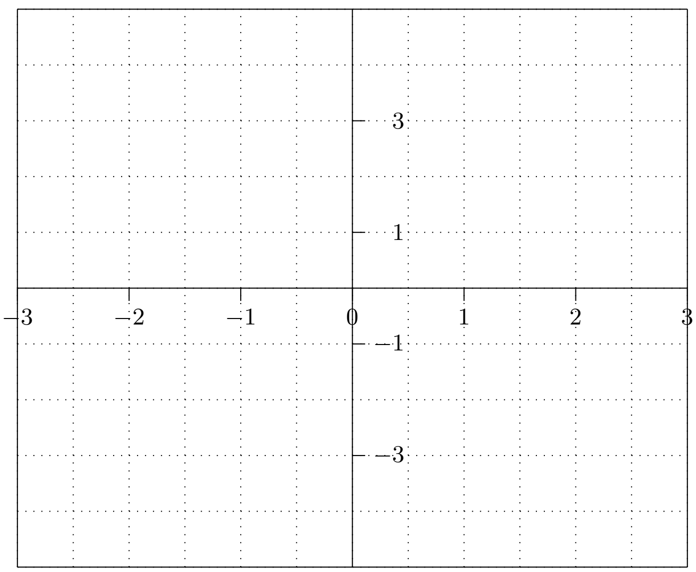
Number Line for $f'$ ________________________________

**3. a)** Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$. Determine the intervals where this function is increasing and where it is decreasing. **Carefully display your answer on a number line as in class.** (Also see online class notes for examples.) WeBWorK Day 29, #1.

To do this which theorem did you use? (See the top of page 1.) ______________________________

**b)** Determine which critical points are local maxima, minima, and which are not extreme and mark this on your number line.
To do this which theorem did you use? (See the top of page 1.) ______________________________

**c)**




Determine the values of $f$ (the original function) for each critical point.

Plot these points, label their coordinates and label whether they are relative extrema.

**Note:** The scales are different on each axis.

Also plot the point when $x = 0$ ($y$-intercept).

**d)** Now using the information in parts (a), (b), and (c) make a sketch of the graph of $f$ *without plotting any more individual points.* The shape of the graph should come from the information in parts (a), (b), and (c).

**e)** Re-use your work from above, as appropriate, to find the **absolute** extrema of $f$ on the closed interval $[-1, 2]$.

Which theorem did you use? (See the top of page 1.) ______________________________

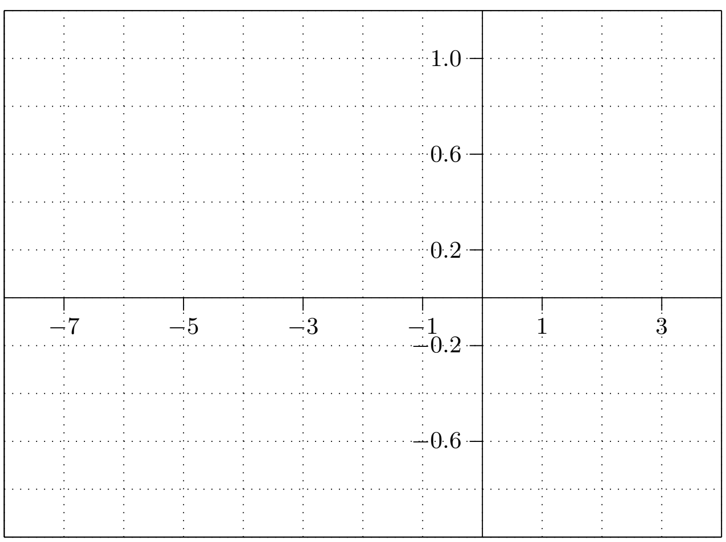
**4. a)** Let $f(x) = \dfrac{2x+4}{x^2+5}$. Determine the intervals where this function is increasing and where it is decreasing. **Carefully display your answer on a number line as in class.** (Also see online class notes for examples.) WeBWorK Day 29, #1.

To do this which theorem did you use? ________________________

**b)** Determine which critical points are local maxima, minima, and which are not extreme and mark this on your number line.

To do this which theorem did you use? ________________________

**c)**

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**d)** Now using the information in parts (a), (b), and (c). make a sketch of the graph of $f$ *without plotting any more individual points.* The shape of the graph should come from the information in parts (a), (b), and (c).

**e)** Re-use your work from above, as appropriate, to find the **absolute** extrema of $f$ on the closed interval $[-2, 2]$.

Which theorem did you use? ________________________