Math 130 Day 30

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

1. CPT: Critical Point Theorem. If f has a local extremum at c, then c is a critical point of f.

2. CIT: The Closed Interval Theorem.
Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

3. The Increasing/Decreasing Test.

a) If $f'(x) > 0$ on an interval, then f is increasing there.

b) If $f'(x) < 0$ on an interval, then f is decreasing there.

4. The First Derivative Test. Let c be a critical number of a continuous function.

a) If f' changes from positive to negative at c, then f has a local maximum at c.

b) If f' changes from negative to positive at c, then f has a local minimum at c.

c) If f' does not change sign at c, then f does **not** have a local extremum at c.

5. The Concavity Test. Assume f is a function whose first and second derivatives exist on an interval I.

a) If $f''(x) > 0$ on the interval, then f is concave up there.

b) If $f''(x) < 0$ on the interval, then f is concave down there.

Practice

Review Section 4.2. Then jump ahead to read Chapter 4.4 on optimization.

a) Read Theorem 4.5 in Chapter 4.2 closely, then try page 257 #49 and 51. This theorem will be important very soon.

b) Basics: Page 257 #53(ez) and 55.

c) Inflections: Page 257 #57, 63, and 64. These are three great problems to get you to simplify derivatives.

d) Matching problem: Page 258 #87. GREAT! Hint: Look to see where the critical points are for each function.

e) Design your own: Page 258: #91 and 93. You can add additional critical points and inflections if needed.

☞ You can download **graph paper** at at the course website at `http://math.hws.edu/~mitchell/Math130S16/index.html`

Hand In

0. WeBWorK set Day30. Due Thursday. The WeBWorK problems are the same as the Hand In problems. Check your work!

Instructions: Graph each of the following functions using the first and second derivative.

- Use number lines for each derivative to organize the information.
- Clearly **mark the critical points and points of inflection with their coordinates on your graph.**
- Note the y-intercept (when $x = 0$) on your graph.
- Make sure that your graph properly illustrates the **increasing** and **decreasing** behavior and the **concavity** of the function.
- **Show all your work**. Do not use a graphing calculator (except to check at the end). You will NOT be able to use a graphing calculator on the exam.
- Be neat and organized. Do each problem (derivatives and graph) on the given page.

☞ See the Model Example on the back of this sheet. Your work should be similar.

0. Model Example. Do a complete graph of $f(x) = \dfrac{x^2}{x^2+1}$ as described in the instructions on page 1. Your work should be similar. Be sure to simplify each derivative.

Math 130 Day 30, Hand In. Name: ______________________

1. Do a complete graph of $f(x) = \frac{1}{4}(x^2 - 9)^2$ as described in the instructions on page 1 (see the model example on page 2). Be sure to simplify each derivative. Check your answer by doing WeBWorK Day 30, #2.

2. Do a complete graph of $f(x) = x^4 + 4x^3 + 10.$ as described in the instructions on page 1 (see the model example on page 2). Be sure to simplify each derivative. Check your answer by doing WeBWorK Day 30, #3.

3. Do a complete graph of $f(x) = \ln(x^2 + 1)$. as described in the instructions on page 1 (see the model example on page 2). Be sure to simplify each derivative. Check your answer by doing WeBWorK Day 30, #4.

4. Close reading: Read Theorem 4.5 in Chapter 4.2 (page 250) closely. Now let $f(x) = \ln x^x$. The domain for this function is $x > 0$. In the space below, answers all of these questions.

a) Show that f has exactly one critical point. (WeBWorK Day 30, #5.)

b) Now determine whether the critical point is a **relative** max or min. Be careful applying the first derivative test. The domain for this function is $x > 0$.

c) What does Theorem 4.5 tell you about this point?

d) Keep a copy of this information for the next assignment.

Example. Find the intervals of concavity and the inflection points for $f(x) = x^4 - 6x^2$. Sketch a graph that includes relative extrema, critical numbers, inflections, increasing/decreasing behavior and concavity.

Solution. (1) Begin with the first derivative to determine critical points, relative extrema, and increasing/decreasing behavior.

$$f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 4x(x - \sqrt{3})(x + \sqrt{3}) = 0$$

CPs: $x =$

f'

Incr/Decr: Evaluate f'

(2) Concavity and Inflections

$f''(x) = 12x^2 - 12 = 12(x^2 - 1)$

Potential Inflections: $x =$

f''

(3) Evaluate $f(x)$ at CPs, Inflects

y intercept (when $x = 0$): $(0, 0)$

x intercept (when $y = f(x) = 0$):

Example. Find the intervals of concavity and the inflection points for $f(x) = 3x^{2/3} - x$. Sketch a graph that includes relative extrema, critical numbers, inflections, increasing/decreasing behavior and concavity.

Solution. (1) Begin with the first derivative to determine critical points, relative extrema, and increasing/decreasing behavior.

$f'(x) = 2x^{-1/3} - 1 =$

CPs: $x =$

f'

Incr/Decr: Evaluate f'

(2) Concavity and Inflections

$f''(x) = -\frac{2}{3}x^{-4/3} = -\frac{2}{3x^{-4/3}}$

Potential Inflections: $x =$

f''

(3) Evaluate $f(x)$ at CPs, Inflects

y intercept (when $x = 0$): $(0, 0)$

x intercept (when $y = f(x) = 0$):

$3x^{2/3} - x = 0 \Longrightarrow x^{2/3}(3 - x^{1/3}) = 0$