Math 130 Day 31

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

1. CIT: The Closed Interval Theorem. Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

2. The Increasing/Decreasing Test.

a) If $f'(x) > 0$ on an interval, then f is increasing there.

b) If $f'(x) < 0$ on an interval, then f is decreasing there.

3. The First Derivative Test. Let c be a critical number of a continuous function.

a) If f' changes from positive to negative at c, then f has a local maximum at c.

b) If f' changes from negative to positive at c, then f has a local minimum at c.

c) If f' does not change sign at c, then f does **not** have a local extremum at c.

4. The Concavity Test. Assume f is a function whose first and second derivatives exist on an interval I.

a) If $f''(x) > 0$ on the interval, then f is concave up there.

b) If $f''(x) < 0$ on the interval, then f is concave down there.

5. SCPT: The Single Critical Point Theorem] Assume that f is differentiable on an interval I (which may be open) and that b is the only critical number of f in I.

a) If f has a local max at b, then f actually has an **absolute max** at b.

b) If f has a local min at b, then f actually has an **absolute min** at b.

Practice

Review Section 4.2. Then jump ahead to read Chapter 4.4 on optimization.

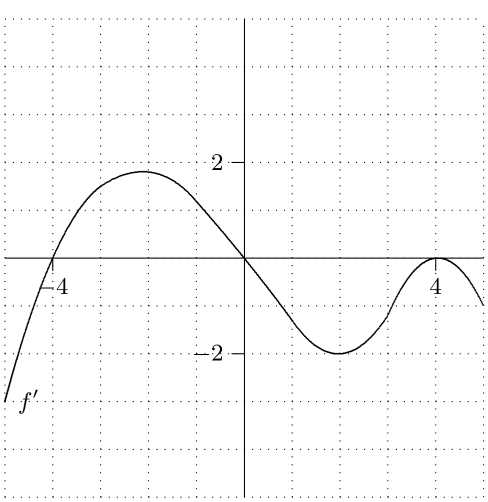
a) Read Theorem 4.5 in Chapter 4.2 closely, then try page 257 #49 and 51. This theorem will be important very soon.

b) Basics: Page 257 #53(ez) and 55.

c) Inflections: Page 257 #57, 63, and 64. These are three great problems to get you to simplify derivatives.

d) Matching problem: Page 258 #87. GREAT! Hint: Look to see where the critical points are for each function.

e) Design your own: Page 258: #91 and 93. You can add additional critical points and inflections if needed.



To the left is the graph of $f'(x)$, the **derivative** of $f(x)$.
This is the graph of $f'(x)$, NOT the graph of $f(x)$. You can read off the values of f' for your 'number line' directly from it. E.g., $f'(2) = -1.8$ (NOT 0)

a) Convert the graph of $f'(x)$ into number line information for $f'(x)$.
Use the number line below the graph. Mark where is $f'(x)$ positive, negative, and 0.

b) Use the number line to classify each critical point of f (loc extrema or not).

c) Use the number line to find where f is increasing and decreasing.

d) Now create a number line for f''. Remember f'' is the derivative (slope) of f'. So where is the slope of f' positive? Negative? 0?
Use this number line to determine concavity and inflections.

e) Sketch the graph of the original function $y = f(x)$. Use the same axes.

You will have to make up values for the function consistent with $f'(x)$.

$f'(x)$

$f''(x)$

Example. Do a complete graph of $f(x) = (x^2-9)^{1/3}$. Be sure to include number lines for the first and second derivatives. Mark the behavior of the function on these number lines. Be sure to mark and label all critical and inflection points and intercepts on your graph.

Solution. (1) Begin with the first derivative to determine critical points, relative extrema, and increasing/decreasing behavior.

$$f'(x) = \frac{1}{3}(x^2-9)^{-2/3}2x = \frac{2x}{3(x^2-9)^{2/3}} = 0$$

CPs: $x =$

f'

Incr/Decr: Evaluate f'

(2) Concavity and Inflections

$$f''(x) = \frac{2\cdot 3(x^2-9)^{2/3} - 2x\cdot 3\cdot \frac{2}{3}(x^2-9)^{-1/3}2x}{9(x^2-9)^{4/3}} \cdot \frac{(x^2-9)^{1/3}}{(x^2-9)^{1/3}} = \frac{6(x^2-9)-8x^2}{9(x^2-9)^{5/3}} = \frac{-54-2x^2}{9(x^2-9)^{5/3}}$$

Potential Inflections: $x =$

f''

(3) Evaluate $f(x)$ at CPs, Inflects

y intercept (when $x=0$): $(0^2-9)^{1/3} \approx -2.080$

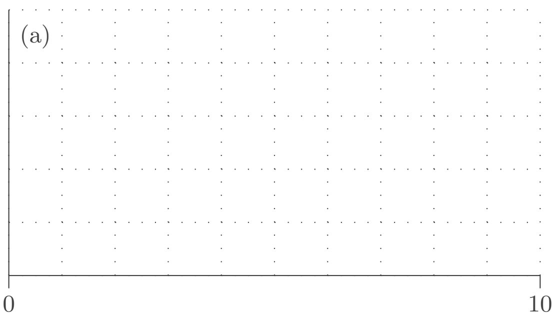
x intercept (when $y = f(x) = 0$): $(x^2-9)^{1/3} = 0$

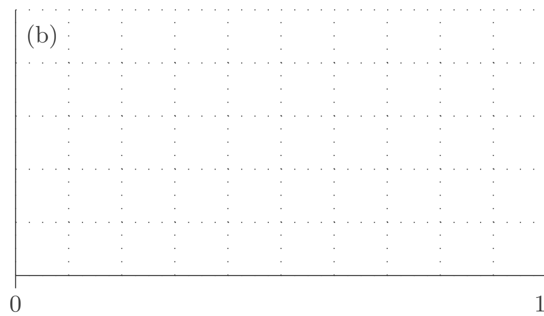
Math 130 Day 31, Lab Hand In. Name: ______________________

1. Do a complete graph of $f(x) = (x-2)e^x$. Be sure to include number lines for the first and second derivatives. Mark the behavior of the function on these number lines. **Be sure to mark and label all critical and inflection points and intercepts on your graph**. Check your answer by doing WeBWorK Day 31, #3.

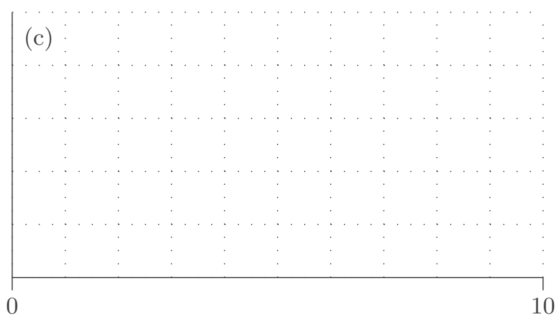
2. a) (Review) Draw the graph of a function on $[0, 10]$ which has no absolute max or explain why this is impossible.

b) Draw a continuous function on $[0, 10]$ which has no absolute min or explain why this is impossible. WeBWorK Day 31, #4.





c) Draw a continuous function on $(0, 10)$ which has no absolute min or explain why this is impossible. WeBWorK Day 31, #5.

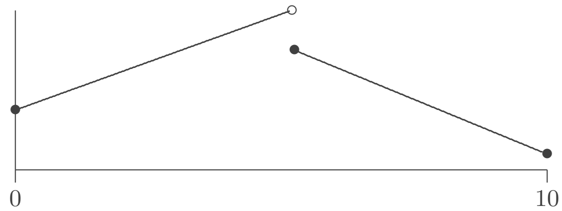


3. a) (Review) Determine the derivative of $f(x) = 8^{x^4-4x^3+1}$.

b) (Review) Determine the derivative of $y = (x^4 + 1)^{\arctan(x^2)}$. What technique is appropriate?

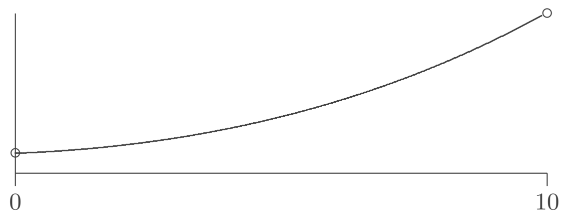
Math 130 Day 31, Answers.

1. a) This is possible if f is not continuous. No absolute max. Make sure the function is defined at all points.



b) Impossible by the EVT a continuous function on a closed interval must have an absolute max and min.

c) This is possible since the interval is not closed. No max or min.



2. a) $f(x) = (x-2)e^x$. Critical numbers:

$$f'(x) = e^x + (x-2)e^x = [1+x-2]e^x = (x-1)e^x = 0. \quad \text{CP} : x = 1.$$

		R Min	
f'	$----$	0	$++++$
	Decr	1	Incr

To do this which theorem did you use? First Derivative Test

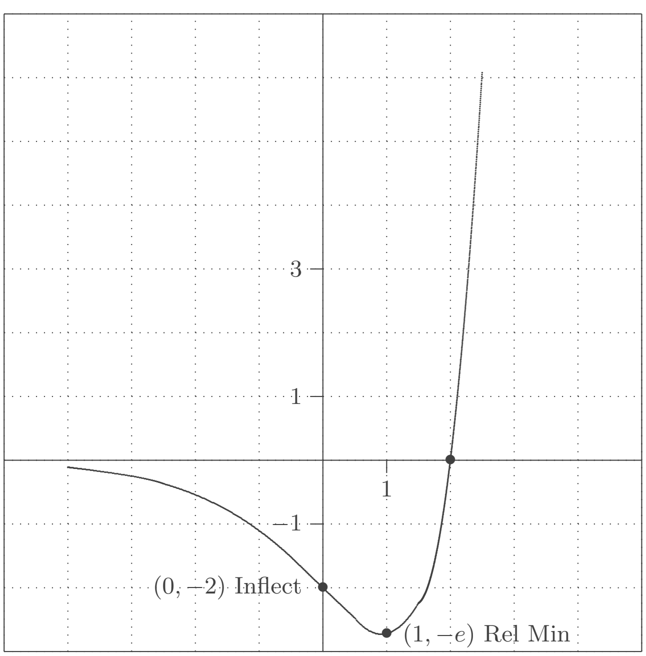
b)

$$f''(x) = e^x + (x-1)e^x = [1+x-1]e^x = x)e^x = 0. \quad \text{CP} : x = 0.$$

		Inflect	
f''	$----$	0	$++++$
	Con Dn	0	Con Up

To do this which theorem did you use? Concavity Test

c)



At the key points:
Relative Min: $f(1) = -e^1 = -e$
Inflect $f(0) = -2e^0 = -2$
x intercept: $(x-2)e^x = 0 \Rightarrow x = 2$

3. a) (Review) Determine the derivative of $f(x) = 8^{x^4-4x^3+1}$.

$$f'(x) = 8^{x^4-4x^3+1} \ln 8(4x^3 - 12x^2).$$

b) (Review) Determine the derivative of $y = (x^4+1)^{\arctan(x^2)}$. What technique is appropriate?

$$\begin{aligned}
\ln y &= \ln(x^4+1)^{\arctan(x^2)} = \arctan(x^2)\ln(x^4+1) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{2x\ln(x^4+1)}{1+x^4} + \frac{4x^3\arctan(x^2)}{1+x^4} \\
\frac{dy}{dx} &= y\left(\frac{2x\ln(x^4+1)}{1+x^4} + \frac{4x^3\arctan(x^2)}{1+x^4}\right) \\
\frac{dy}{dx} &= (x^4+1)^{\arctan(x^2)}\left(\frac{2x\ln(x^4+1) + 4x^3\arctan(x^2)}{1+x^4}\right)
\end{aligned}$$