Math 130 Day 32

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

1. ☞ **CIT: The Closed Interval Theorem.** Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

2. ☞ **SCPT: The Single Critical Point Theorem.** Assume that f is a continuous function on an interval (open, closed, or half-open) I. Assume that c is the only critical number of f.
   a) If f has a relative maximum at c, then f has a absolute maximum at c.
   b) If f has a relative minimum at c, then f has a absolute minimum at c.

Practice

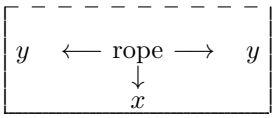
Read section 4.4; concentrate on the examples. Practice using the homework and finish the classwork. Optimization problems require lots of practice and will be on the exam.

1. **a)** These problems are straightforward. Make sure you can do them: Page 275ff #5, 9, 11, 15. A bit harder, but still very doable: #17, 21(a). Now try #29. **Extra Credit**: Page 262 #26.

Extra Practice

Use and label the steps we discussed in class to solve these problems. Pay attention to the domain! Justify your answer with the CIT or SCPT (say which applies and why).

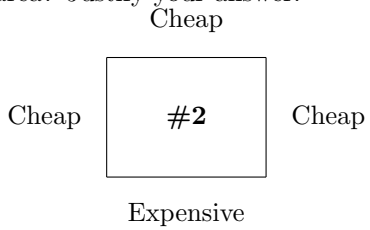
1. A rectangular garden contains an area of 200 m^2 and is roped off on three sides (see below). What dimensions minimize the length of the rope? **Important**: Indicate the domain of the function and **justify** your answer with the CIT or SCPT above, as appropriate. WeBWorK set Day 32.

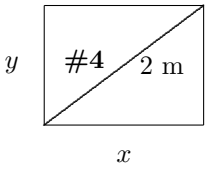


2. A cheap homeowner wants to fence in a rectangular plot of 300 m^2. He will put expensive fence costing \$5 per meter along the front and cheap stuff costing \$1 per meter along the sides and back. What dimensions minimize the cost? (See below.)

3. Find the absolute extreme points of $f(x) = x\sqrt{4 - x^2}$ on the interval $[0, 2]$. Be careful calculating $f'(x)$.

4. **Extra credit** (Alfred Croteau (S' 04)): A rectangle has a diagonal of 2 meters. What dimensions of the rectangle produce the largest area? Justify your answer.





Applied Optimization (Max-Min) Problems

Example A: A soup can is to hold 18 oz. What dimensions for the radius and height of the can *minimize* the cost (i.e., the materials) for the can. Here, **optimize** means minimize.

Example B: What tuition optimizes (maximizes) the revenue for HWS?

Overview

Such problems can be readily solved using the following analysis. Don't think of this as a recipe to memorize, rather try to understand the goal of each problem and then make use of the theory we have developed to solve the problem. Remember your solution is an essay or argument that in which you try to convince your reader (using evidence) that you have found a solution.

1. Identify the problem: Find the *absolute* max or min of a quantity Q [that is usually expressed as a function of two variables, which we call x and y].
2. Subject to a *constraint:* Usually a constant $k =$ a simple function of x and y.
3. Use the constraint to eliminate a variable, e.g., 'solve for y.'
4. Write Q as a function of one variable, $Q(x)$, and state its *domain*, which is often determined by the constraint equation. I cannot overemphasize determining the domain of $Q(x)$ in this process.
5. Find the absolute max or min of Q on its domain. The method you use will be determined, in part, by the domain. If the domain is a closed interval, you can use the CIT. If it is not closed, then you must hope that there is only one critical number so that you can use the SCPT. (The SCPT can also be used on a closed interval if there is a single critical number.) **Justify your answer in a sentence.**

Notice that only step 5 do we actually do any calculus.

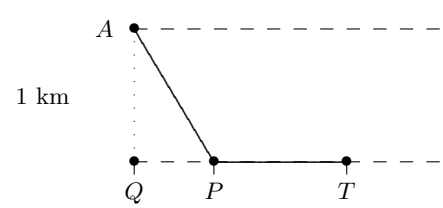
Classwork Examples: Optimization

Carefully justify your answer using an appropriate theorem (CIT or SCPT, say which applies and why). Answers without justification will lose substantial credit. Your work should be a 'mini-essay' that demonstrates your critical thinking about the given problem. Pay attention to the domain!

1. A real estate developer wishes to enclose a rectangular plot by a fence and then divide it into two plots by another fence parallel to one of the sides. What are the dimensions of the largest area that can be enclosed by using a total of 1800 meters of fencing?

2. Find the dimensions of the box with a square base that has a volume of 512 in^3 and has the *smallest* surface area (4 sides and 2 ends).

3. A cable television company has its master antenna located at point A on the bank of a straight river 1 kilometer wise (Figure 1 above). It is going to run a cable from A to a point P on the opposite bank of the river and then straight along the bank to a town T situated 3 kilometers downstream from A. It costs \$15 per meter to run the cable under the water and \$9 per meter to run the cable along the bank. What should be the distance from P to T in order to minimize the total cost of the cable?



4. A child's sandbox is to be made by cutting equal squares from the corners of a square sheet of aluminum and turning up the sides. If each side of the sheet of galvanized iron is 2 meters long, what size squares should be cut from the corners to maximize the volume of the sandbox?

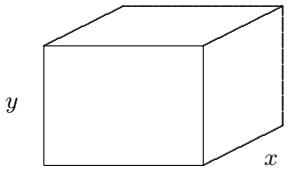
5. Advertising fliers are to be made from rectangular sheets of paper that contain 400 square centimeters of printed message. If the margins at the top and bottom are each 5 centimeters and the margins at the sides are each 2 centimeters, what should be the dimensions of the fliers if the total page area is to be a minimum?

Hand In. Name: _____

Do WeBWorK Set Day 32 due Tuesday night.

1. (Similar to WeBWorK Set Day 32 #1). A small box with a **square base** and **no top** is to hold a volume of 256 cc. What dimensions for the box will minimize the materials for the 4 sides and bottom? Justify your answer. Solution: Using the guidelines

a) Optimize (Minimize?, Maximize?):



b) Constraint: Subject to:

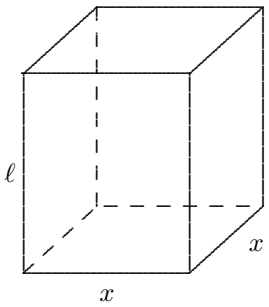
c) Eliminate (if necessary):

d) Rewrite: Domain:

e) Solve and **Justify:**

2. (WeBWorK Set Day 32 #4) US Postal regulations `usps.com/consumers/domestic.htm` state: "Priority Mail is used for documents, gifts, and merchandise. The maximum size is 108 inches or less in combined length (ℓ) and distance around the thickest part (Girth, G)." Find the dimensions of the box with square base of largest volume that can be sent Priority Mail. Justify your answer. Solution: Using the guidelines

a) Optimize (Minimize?, Maximize?):



b) The Girth is the *perimeter around the square end*, not the area of the square end (see figure). So

Subject to Girth + length = 108. So 108 = _____ $+ \ell$

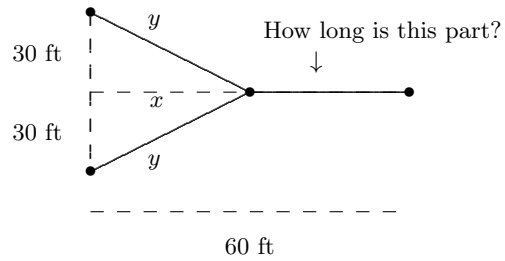
c) Eliminate ℓ.

d) Rewrite: $V =$ Domain:

e) Solve and **Justify:**

3. A cable tv guy has to run cable from where it enters the house to two separate tv's as shown. At what point x should he split the cable feed to minimize the amount of wire used? (See figure. Note the dashed line labeled 60ft. What does that make the length of the unlabeled edge?) Solution: Using the guidelines

a) Optimize (Minimize?, Maximize?):



b) Constraint: Subject to:

c) Eliminate (if necessary):

d) Rewrite: Domain:

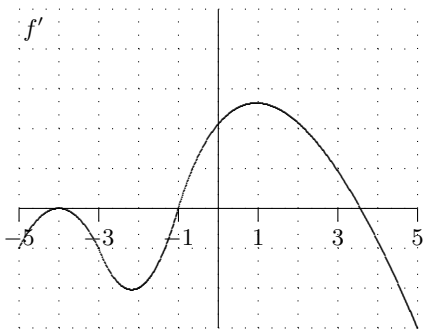
e) Solve and **Justify:**

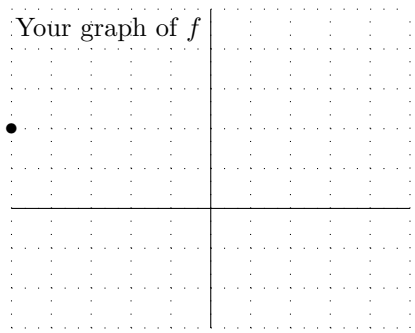
4. Below is the graph of $f'(x)$, **not** $f(x)$.

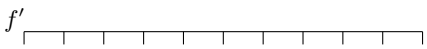
a) Translate this graphical information into number line information for f' and f''. For example, where f' is above the x-axis, f' is positive. Where the graph of f' is decreasing, that's where f'' is negative, where f' has a horizontal tangent, that's where $f'' = 0$. Here are some specific examples

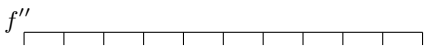
- At $x = 3.5$: Note $f'(3.5) = 0$ (the graph crosses the x-axis and $f''(3.5)$ is negative because $f'(x)$ is decreasing there. Mark each on the corresponding number line.
- At $x = 1$: Note $f'(1)$ is positive (the graph is above the the x-axis and $f''(1) = 0$ because $f'(x)$ has a horizontal tangent there. Mark each on the corresponding number line.
- Continue in the same manner. You should be able to fill in each number line where it is 0, +, and −.

b) Based on the number line information (relative extrema, inflections, etc.), draw a possible graph of the original function $y = f(x)$. Start your graph at the indicated point •.









5. Extra Credit: Like WeBWorK Set Day 32 #5: The manager of a large apartment complex knows from experience that 80 units will be occupied if the rent is 372 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 6 dollar increase in rent. Similarly, one additional unit will be occupied for each 6 dollar decrease in rent. What rent should the manager charge to maximize revenue?