Math 130 Day 37

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Practice, Etc

1. Review Chapter 4.7 on L'Hopital's Rule. See the **notes online** with more examples. Begin Chapter 4.8: Antiderivatives.

  a) These are quick: Try page 307 #15, 19, 22(Ans: 4/7), 23, 31, 33.

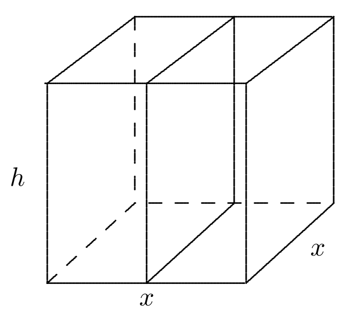
  b) Now try page 307 #37, 41, and 49. Try $\lim_{x\to\infty} x^2e^{-x}$, $\lim_{x\to\infty} x\tan(\frac{1}{x})$ $\lim_{x\to 0^+} x^2\ln x$. Answers: 0, 1, 0.

2. Do the Optional Re-do problems on Page 4.

3. Fill out all your course evaluations.

Answers to Test 3

1. A small cardboard box is to contain a volume of 6,400 cm^3. The box has a square base and inside it has an additional vertical **cardboard divider** as shown [to make two compartments]. Find the dimensions of the box that minimize the amount of materials used for the sides, top, bottom, and divider. Fully **justify** your answer.



`Minimize` $A = 2x^2 + 5xh$ `(top + bottom) + (4 sides + divider)`

`Constraint:` $V = x^2h = 6400$ `cm`^{3}

`Eliminate:` $h = \dfrac{6400}{x^2}$. `Notice that` x `cannot be` 0`, but using any positive value for` x `we can solve for` h`. So the domain is` $(0,\infty)$.

`So:` $A = 2x^2 + 5x\cdot\dfrac{6400}{x^2} = 2x^2 + \dfrac{32000}{x}$ `on` $(0,\infty)$. `Therefore:`

$$A' = 4x - \frac{32000}{x^2} = 0 \Rightarrow 4x = \frac{32000}{x^2} \Rightarrow x^3 = 8000 \Rightarrow x = 20.$$

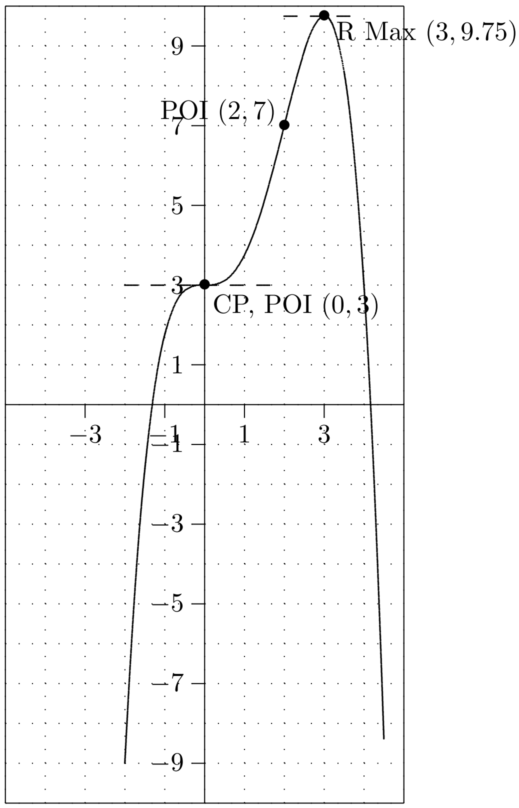
`Using the First Derivative Test, there's a local min at` $x = 20$

RMin
f': $- - - -$ 0 $+ + + +$ (0 ... 20)

`Notice that there's just a single critical point in the domain. So by SCPT, the relative min at` $x = 20$ `is an absolute minimum. And` $h = \frac{6400}{(20)^2} = 16$.

2. `To find the absolute maximum and minimum values of` $f(x) = x^2e^{-x}$ `on the interval` $[-2, 1]$ `use the CIT.`
`First find the critical points by taking the derivative.`
$f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{x}(2 - x) = 0$ `at` $x = 0$ `(and` $x = 2$ `which is outside the domain).`
`The CIT says to check` f `at the critical and end points and pick the extreme values.`
`At the critical point:` $f(0) = 0$.
`At the endpoints:` $f(-2) = 4e^{-(-2)} = 4e^2 \approx 29.556$.
$f(1) = e^{-1} \approx 0.368$.
`By the CIT, the Abs Max is` $4e^2$ `which occurs at` $x = -2$ `and the Abs Min is 0 which occurs at` $x = 0$.

3. Graph $f(x) = x^3 - \frac{1}{4}x^4 + 3$. Show everything.



First find and classify the critical points.

$f'(x) = 3x^2 - x^3 = x^2(3 - x) = 0 \Rightarrow$ CPs: $x = 0,\ 3$.

		Neither		RMax	
f'	$++++$	0	$++++$	0	$----$
	Inc	0	Inc	3	Dec

Next determine the potential POIs

$f''(x) = 6x - 3x^2 = 3x(2 - x) = 0 \Rightarrow$ at $x = 0,\ 2$ (Potential POIs).

		POI		POI	
f''	$---$	0	$+++$	0	$---$
	Con Dn	0	Con Up	2	Con Dn

Plot Key Points (CPs, POIs, and Intercepts)

$f(0) = 3$ (also the y-intercept)

$f(3) = 27 - \frac{81}{4} + 3 = 9.75$,

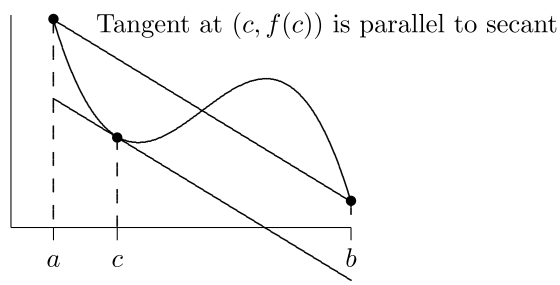
$f(2) = 8 - 4 + 3 = 7$.

x-intercept

$f(x) = x^3 - \frac{1}{4}x^4 + 3 = 0$ not easily solved.

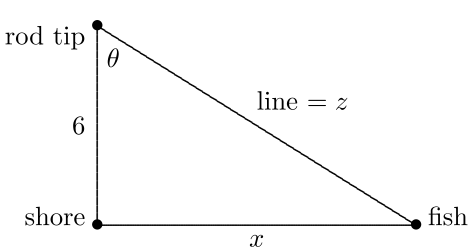
4. a) MVT: The Mean Value Theorem. Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there is some point c between a and b so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$



b) `c is a critical number of f if c is an interior point of the domain of f and either` $f'(c) = 0$ `or` $f'(c)$ `DNE. Also acceptable: c is a critical number of f if f is defined at c and either` $f'(c) = 0$ `or` $f'(c)$ `DNE.`

5. A child is fishing on a shore line. She catches a fish which swims directly away from the shore at a rate of 4 ft/s. Assume that the rod tip is 6 ft above the shore. How is the angle θ between the rod and the fishing line changing when the fish is 8 ft from shore? (See diagram. Turn the page sideways to get a more familiar view.)



`Method 1:` Given $\dfrac{dx}{dt} = 4$ ft/s. (How fast the fish is moving from the shore.)

Find $\left.\dfrac{d\theta}{dt}\right|_{x=8}$. (How fast the angle θ is changing when $x = 8$.)

Relation: Find a relation between x and θ.

When using a trig relation, use a constant side in the trig relation if possible.

This will simplify taking the derivative.

$\tan\theta = \dfrac{x}{6}$, which means that $\theta = \arctan\left(\frac{x}{6}\right)$.

Rate: $\dfrac{d\theta}{dt} = \dfrac{1}{1 + \left(\frac{x}{6}\right)^2} \cdot \dfrac{1}{6} \cdot \dfrac{dx}{dt}$.

Evaluate: $\left.\dfrac{d\theta}{dt}\right|_{x=8} = \dfrac{1}{1 + \left(\frac{8}{6}\right)^2} \cdot \dfrac{1}{6} \cdot (4) = \dfrac{1}{\frac{100}{36}} \cdot \dfrac{4}{6} = \dfrac{6}{25} = 0.24$ rad/s

Method 2: Relation: $\tan\theta = \dfrac{x}{6}$

Rate: $\sec^2\theta \dfrac{d\theta}{dt} = \dfrac{1}{6}\cdot\dfrac{dx}{dt}$.

Evaluate. When $x = 8$, hypot $= z = \sqrt{6^2+8^2} = 10$, so $\sec\theta = \dfrac{10}{6}$. Substitute to get

$$\left(\frac{10}{6}\right)^2 \frac{d\theta}{dt}\bigg|_{x=8} = \frac{1}{6}\cdot(4) \Rightarrow \frac{d\theta}{dt}\bigg|_{h=8} = \left(\frac{6}{10}\right)^2\cdot\frac{4}{6} = \frac{6}{25} = 0.24 \text{ rad/s}$$

6. a) Determine the derivative of $f(x) = 8^{x^2\cos x}$

Solution: Avoid logarithmic differentiation. Use $\dfrac{d}{dx}[b^u] = b^u(\ln b)\dfrac{du}{dx}$. Here $u = x^2\cos x$.

$$f'(x) = 8^{x^2\cos x}(\ln 8)\overbrace{(2x\cos x - x^2\sin x)}^{\text{Product Rule}}$$

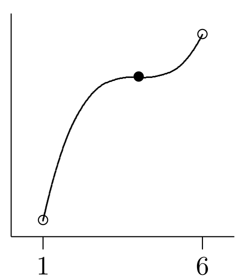
b) Determine the derivative of $y = (2+\sin x)^{4x^3}$.
Solution: Use logarithmic differentiation. Take the logs of both sides:

$$\begin{aligned}\ln y &= \ln(2+\sin x)^{4x^3}\\ \ln y &= 4x^3\ln(2+\cos x)\end{aligned}$$

Now take the derivative

$$\begin{aligned}\frac{1}{y}\frac{dy}{dx} &= 12x^2\ln(2+\cos x) + 4x^3\left(\frac{\cos x}{2+\sin x}\right)\\ \frac{dy}{dx} &= y\left[12x^2\ln(2+\sin x) + \frac{4x^3\cos x}{2+\sin x}\right]\\ \frac{dy}{dx} &= (2+\cos x)^{4x^3}\left[12x^2\ln(2+\sin x) + \frac{4x^3\cos x}{2+\sin x}\right]\end{aligned}$$

7.

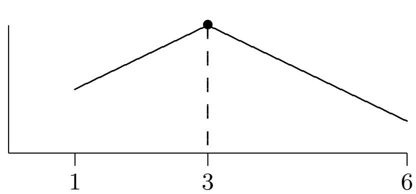


a) Draw the graph of a continuous function defined on $(1,6)$ which has no absolute max, or explain why this is impossible.

Possible: The function graphed to the left has no max or min because the interval is not closed.

b) Draw the graph of a differentiable function on $[1,6]$ which has no absolute max, or explain why this is impossible.

Impossible. Since f is differentiable, it is continuous. So by EVT or CIT it must have both an absolute max and min on the closed interval.



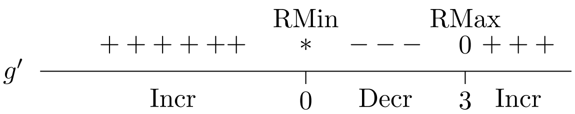
c) Draw the graph of a continuous function on $[1,6]$ for which has a relative max at $x=3$ but for which $f'(3)\neq 0$.

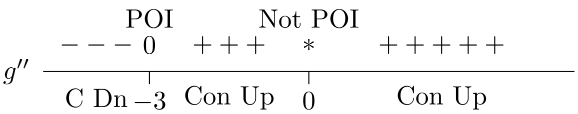
Possible; put a corner there.

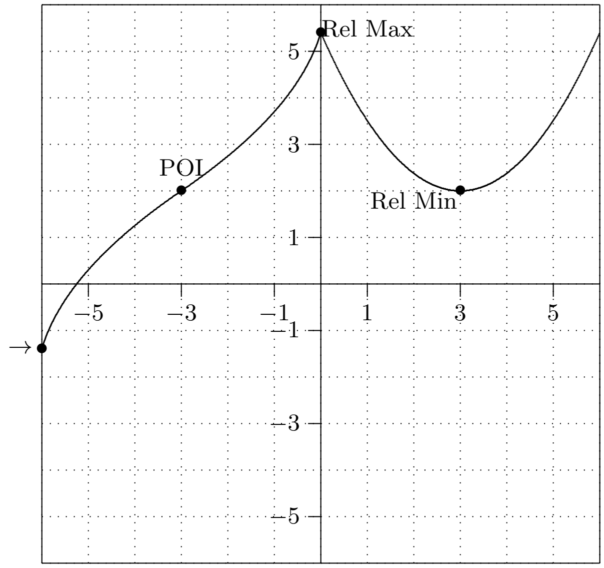
d) True or False: If f is a differentiable function on $(1,6)$ that has a single critical point, then the critical point is either an absolute max or absolute min. (Explain or illustrate.)

False: The function graphed in part (a) has a single critical point at • and it is neither an absolute max or min. Note the SCPT does not apply. To use the SCPT, you must have a continuous function (we do since f is differentiable) and a single critical point (we do), AND you need to know that the critical point IS a local max or local min (say by using the First Derivative Test). That might not be the case, as in the graph in part (a).

8. (7 pts) Sketch a *continuous* function that would have first and second derivatives like those given below. On the number lines indicate where the function is increasing, decreasing, concave up, and concave down; indicate all relative extrema, and inflections. Indicate on your graph which points are local extrema and which are inflections. $*$ indicates that the derivative does not exist at the point (though the original function does). Start your graph at the point $\bullet$ indicated.



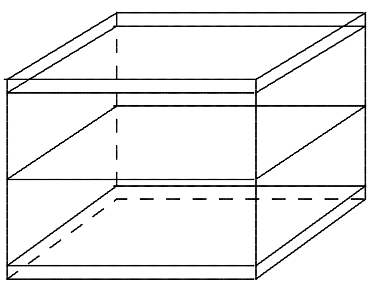




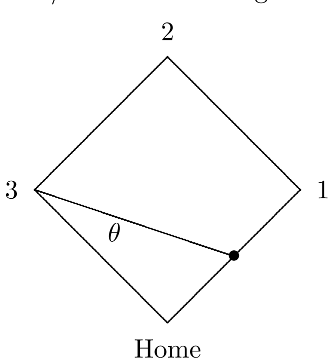
Optional: Max-Min and Related Rate Re-dos.

These two problems may be downloaded on a separate sheet at our website.

1. A small cardboard box is to contain a volume of 2,500 cm^3. The box has a **square base** and is constructed with **two layers** of cardboard on the top and bottom. Inside it has an additional horizontal cardboard divider as shown [to make two compartments]. Find the dimensions of the box that minimize the amount of materials used for the sides, top, and bottom. Carefully justify your answer.



2. A baseball diamond is a square with 90 ft sides. Derek Jeter hits the ball and runs towards first base at a speed of 24 ft/s. How is the angle θ changing at this moment?



Math 130 Dy 37, Hand In. Name: ______________________

0. Do WeBWorK Set Day 37, due Thursday night. Finish WeBWorK Set Day 35–36 tonight. Optional Review: Do the two Re-dos on the previous page.

1. Use L'Hopital's rule, if appropriate, to determine each of these limits after determining the indeterminate form of the limit. Show your work.

a) $\lim_{x \to 0} \frac{\tan x}{\sin x} =$

b) $\lim_{x \to 0} \frac{\sin 5x}{\sin 2x} =$

c) $\lim_{x \to +\infty} \frac{x^2}{e^{2x}} =$

d) $\lim_{x \to +\infty} \frac{\ln 2x}{x^2} =$

e) $\lim_{x \to 1} \frac{6^x + 4^x - 10}{e^{x-1} - 1} =$

f) If we get this far: $\lim_{x \to 0^+} x^2 \ln x^2 =$

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2. Draw a detailed graph of $f(x) = \frac{x}{e^x}$. Determine the domain. Include all extrema, inflections, and asymptotes (both VA and HA, if any) and any appropriate 'end behavior.' Indicate any intercepts as well. Show all work. **This is the same as WeBWorK Problem 1 on Set Day 37 which you should do first!** Be especially careful evaluating $\lim_{x \to -\infty} \frac{x}{e^x}$. It is NOT of the indeterminate form $\frac{\infty}{\infty}$. What is it?

Optional: Max-Min and Related Rate Re-dos. Name: ____________________________

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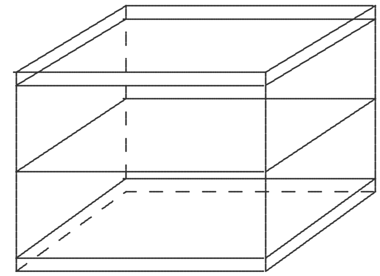

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