Math 130 Day 39

Practice

Since antidifferentiation undoes or reverses the differentiation process, each derivative rule can be viewed as an antiderivative rule. We will use the MVT to show that two antiderivatives of the same function differ by at most a constant.

1. **a)** Reread: Section 4.9 on Antiderivatives.

   b) Practice: Page 327ff: #11–49, 55, and 57 odd. Try at least some. The later ones are more interesting.

2. Begin to review for the final. Start with Labs and the previous Practice Tests (all on line with answers).

3. (**Review: Answers on the last page.**) Determine the **horizontal asymptotes** of $y = f(x) = \dfrac{6x-1}{\sqrt{4x^2-2x+2}}$.

4. **Review:** Determine the **horizontal** and **vertical** asymptotes of $f(x) = \dfrac{x^2+x-2}{2x^2-2x}$. (Answers on the last page.)

5. **Review:** Do a complete graph (critical points, extrema, inflections, increasing, decreasing, concavity, vertical and horizontal asymptotes) of $y = f(x) = \dfrac{x^2+3x-3}{x^2}$. (Answers online at course website.)

Memorize the Antiderivative Rules

Antidifferentiation Reverses Differentiation

Each derivative rule has a corresponding antidifferentiation rule.

Differentiation	Antidifferentiation	
$\frac{d}{dx}(c) = 0$	$\int 0\,dx =$	
$\frac{d}{dx}(kx) = k$	$\int k\,dx =$	
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx =$	$,\ n \neq -1$
$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$	$\int x^{-1}\,dx = \int \frac{1}{x}\,dx =$	
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx =$	
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx =$	
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx =$	
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx =$	
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx =$	
$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx =$	
$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx =$	
$\frac{d}{dx}(b^x) = b^x \ln b$	$\int b^x \ln b\,dx =$	

Variations and Generalizations

Notice what happens when we use ax instead of x in some of these functions. We multiply by a when taking the derivative, so we have to divide by a when taking the antiderivative.

Differentiation	**Antidifferentiation**
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}\,dx =$
$\frac{d}{dx}(\sin ax) = a\cos ax$	$\int \cos ax\,dx =$
$\frac{d}{dx}(\cos ax) = -a\sin x$	$\int \sin ax\,dx =$
$\frac{d}{dx}(\tan ax) = a\sec^2 ax$	$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + c$
$\frac{d}{dx}(\sec ax) = a\sec ax\tan ax$	$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax + c$
$\frac{d}{dx}(\arcsin(\frac{x}{a})) = \frac{1}{\sqrt{a^2-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin(\frac{x}{a}) + c$
$\frac{d}{dx}(\arctan(\frac{x}{a})) = \frac{a}{a^2+x^2}$	$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan(\frac{x}{a}) + c$

6. Here are a few examples.

a) $\displaystyle\int 4x^6\,dx =$

b) $\displaystyle\int 3x^{3/5}\,dx =$

c) $\displaystyle\int (2\sin x + 12\cos x)\,dx =$

d) $\displaystyle\int e^{z/2}\,dz =$

e) $\displaystyle\int \frac{1}{16+x^2}\,dx =$

f) $\displaystyle\int \cos(4x)\,dx =$

g) $\displaystyle\int (4e^x - 12\sec^2 x)\,dx =$

h) $\displaystyle\int \frac{6}{1+t^2}\,dt =$

i) $\displaystyle\int \left(\frac{8}{x} - \frac{2}{\sqrt{1-x^2}}\right)dx =$

7. Simplify or rewrite the integrands if necessary before anti-differentiating.

a) $\displaystyle\int 4\sqrt[3]{x^7}\,dx =$

b) $\displaystyle\int \frac{4}{\sqrt[3]{x^7}}\,dx =$

c) $\displaystyle\int \frac{1}{2s^5}\,ds =$

d) $\displaystyle\int (t^2-3)^2\,dt =$

e) $\displaystyle\int \frac{t^2+t+1}{\sqrt{t}}\,dt =$

f) Now try $\int 4e^{-s}\,ds$. **Check your answer.** You may need to correct your first guess!

Answers from the problems on the other side.

3. a) Remember that $\sqrt{x^2} = |x|$. $\displaystyle\lim_{x\to+\infty}\frac{6x-1}{\sqrt{4x^2-2x+2}} = \lim_{x\to+\infty}\frac{6x}{\sqrt{4x^2}} = \lim_{x\to+\infty}\frac{6x}{|2x|} = \lim_{x\to+\infty}\frac{6x}{2x} = 3$

b) $\displaystyle\lim_{x\to-\infty}\frac{6x-1}{\sqrt{4x^2-2x+2}} = \lim_{x\to-\infty}\frac{6x}{\sqrt{4x^2}} = \lim_{x\to-\infty}\frac{6x}{|2x|} = \lim_{x\to-\infty}\frac{6x}{-2x} = -3$
So $y = 3$ and $y = -3$ are the horizontal asymptotes

4. a) HA: $\lim_{x\to-\infty}\frac{x^2+x-2}{2x^2-2x} = \lim_{x\to-\infty}\frac{x^2}{2x^2} = \frac{1}{2}$ and $\lim_{x\to+\infty}\frac{x^2+x-2}{2x^2-2x} = \lim_{x\to+\infty}\frac{x^2}{2x^2} = \frac{1}{2}$. HA: $y = \frac{1}{2}$.

b) VA: Check at $x = 1$ and -1.

$\displaystyle\lim_{x\to1}\frac{x^2+x-2}{2x^2-2x} = \lim_{x\to1}\frac{(x-1)(x+2)}{2(x-1)(x+1)} = \lim_{x\to1}\frac{x+2}{2(x+1)} = \frac{3}{4}$. So $x = 1$ is NOT a VA.

$\displaystyle\lim_{x\to-1^+}\frac{x^2+x-2}{2x^2-2x} = \lim_{x\to-1^+}\frac{(x-1)(x+2)}{2(x-1)(x+1)} = \lim_{x\to-1^+}\frac{x+2}{2(x+1)} \to \frac{1}{0^+} = +\infty.$

$\displaystyle\lim_{x\to-1^-}\frac{x^2+x-2}{2x^2-2x} = \lim_{x\to-1^-}\frac{x+2}{2(x+1)} \to \frac{1}{0^-} = -\infty.$ So $x = -1$ is a VA.

5. NID: $x = 0$. (Domain $(-\inf, 0) \cup (0, \inf)$.)

VA: $\displaystyle\lim_{x \to 0^+} \frac{x^2 + 3x - 3}{x^2} \to \frac{-3}{0^+} = -\infty$ and $\displaystyle\lim_{x \to 0^-} \frac{x^2 + x - 2}{x^2} \to \frac{-3}{0^-} = -\infty$; so VA at $x = 0$.

HA: $\displaystyle\lim_{x \to +\infty} \frac{x^2 + 3x - 3}{x^2} = \lim_{x \to +\infty} \frac{1 + \frac{3}{x} - \frac{3}{x^2}}{1} = 1$ so HA at $y = 1$.

HA: $\displaystyle\lim_{x \to -\infty} \frac{x^2 + 3x - 3}{x^2} = \lim_{x \to -\infty} \frac{1 + \frac{3}{x} - \frac{3}{x^2}}{1} = 1$ so HA at $y = 1$.

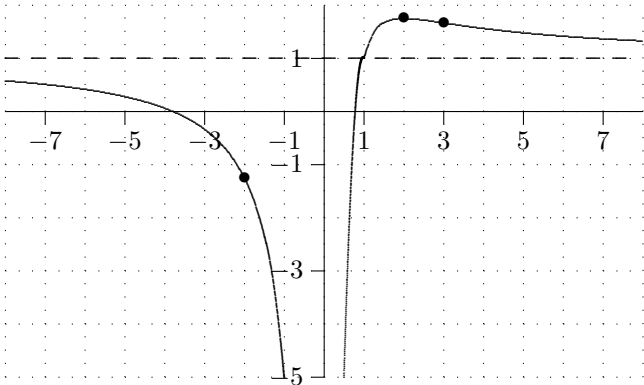$f'(x) = \dfrac{(2x + 3)x^2 - (x^2 + x3 - 3)2x}{x^4} = \dfrac{(2x + 3)x - (x^2 + 3x - 3)2}{x^3} = \dfrac{6 - 3x}{x^3} = 0$ at $x = 2$

$f''(x) = \dfrac{-3x^3 - (6 - 3x)3x^2}{x^6} = \dfrac{-3x - (6 - 3x)(3)}{x^4} = \dfrac{6x - 18}{x^4} = \dfrac{6(x - 3)}{x^4} = 0$ at $x = 3$.

Evaluate f at key points. $f(2) = \frac{7}{4}$, $f(3) = \frac{5}{3}$. Since there are no key points on the left half of the graph, evaluate f at some point where x is negative, say $f(-2) = -\frac{5}{4}$

f' 　$-- \text{NID}$　$+++$　$\overset{\text{RMax}}{0}$　$--$

　$\underset{0}{\text{Dec}}$　Inc　$\underset{2}{}$ Dec

f'' 　$-- \text{NID}$　$---$　$\overset{\text{INF}}{0} +$

　$\underset{0}{\text{CD}}$　CD　$\underset{3}{}\text{CU}$

The inflection is almost imperceptible

Math 130 Homework: Hand in Monday. Name: _____

Do WeBWorK set Day 39. (Due Tuesday.) There are about 25 antiderivatives to try.

1. Eight from your text: Do page 328–29. Use the rules on pages 322 and 324 or on pages 1 and 2. **Check your answers by differentiating.** You may need to correct your first guess! NOTE: Write out the original question, not just the answer.

a) #24

b) #26

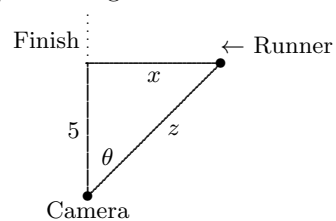**c)** #28 (multiply first)

d) #30

e) #32

f) #34(rewrite as a fractional power)

g) #36(divide first)

h) #48.

2. (Review) A TV camera is located 5 meters from the finish line as shown. How fast must the camera turn (i.e., how is the angle changing) when the runner is 15 meters from the finish line and moving at 10m/s? See figure below.

Finish ← Runner

x

5

z

θ

Camera

Math 130, Day 39. Answers

1. Eight from your text: Do page 328–29. **Check your answers by differentiating.** NOTE: Write out the original question, not just the answer.

   a) #24

   b) #26

   c) #28 (multiply first)

   d) #30

   e) #32

   f) #34 (rewrite as a fractional power)

   g) #36 (divide first)

   h) #48.

2. Given $\frac{dx}{dt} = -10$ m/s (distance to finish line is decreasing!).

   Find $\frac{d\theta}{dt}\Big|_{x=15}$. Relation: $\tan\theta = \frac{x}{5}$.

   Rate-ify: $\sec^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$ or $\frac{d\theta}{dt} = \frac{1}{5\sec^2\theta}\frac{dx}{dt} = \frac{\cos^2\theta}{5}\frac{dx}{dt}$.

   Evaluate: Use the triangle.

   When $x = 15$, the hypotenuse is $\sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$. So $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{5\sqrt{10}} = \frac{1}{\sqrt{10}}$. So

   $$\frac{d\theta}{dt}\Big|_{x=5} = \frac{\cos^2\theta}{5}\frac{dx}{dt} = \frac{\left(\frac{1}{\sqrt{10}}\right)^2}{5}(-10) = \frac{\frac{1}{10}}{5}(-10) = -0.2\,\text{rad/s}.$$