Math 130 Day 42

Office Hours (LN 301/301.5): M 3:30-4:30, Tu 11:00-1:00, W 12:15-1:15, F 1:30-2:30. Other times by appointment. **Math Intern:** Sun through Thurs: 3:00–6:00, 7:00–10:00pm. **Website:** Use the links at the course homepage on **Canvas** or go to my course Webpage: `http://math.hws.edu/~mitchell/Math130F16/index.html`.

Review, Office Hours, Final Exam

1. **a)** I will have my usual Office Hours: Monday 2:30 to 4:00 PM and Tues 2:00 to 3:30 PM.

b) Optional Review/Lab Session: Monday December 12 at 10:30–12:00 in Gulick 206A.

c) The **Math Intern** will be holding regular office hours in Lansing 310.

2. Website: `http://math.hws.edu/~mitchell/Math130F16/index.html`. All of the earlier course materials are here including Practice Problems for the previous exams, course notes, course assignments and answers, and Labs with answers.

3. **Final Exam**: Wednesday, Dec 14 at 8:30 (Section 2) and 1:30 (Section 1) in Napier 201.

4. **Practice Problems** now online at the address above. The answers will be available on the weekend.

5. The **Final Exam** is cumulative. Use the labs and the previous tests and practice tests as a guide. I will post most of the answers to the **Practice Review** on line.

a) You should know the following definitions and how to use them: the definition of a continuous function, the limit definition of the derivative, the definition of an antiderivative, the definition of a horizontal or a vertical asymptote, the definition of a critical point.

b) You should know which functions are polynomials, rational, and which are neither.

c) You should know (word for word) the MVT. You should know how to use the EVT, CIT, SCPT, IVT. You should know the relationship between differentiable and continuous functions.

d) Inverse functions and derivatives: Review how we found the derivatives of $\arctan x$, $\arcsin x$, and $\ln x$ (as the inverse of e^x). Use of triangles to calculate certain inverse trig expressions.

e) Know all limit rules and when to use factoring, conjugates, known limits (e.g., with trig functions), and limits at infinity). Know how to use l'Hopital's rule and when it applies (to which indeterminate forms).

f) Know all derivative rules including trig, inverse trig functions, natural log, (general) exponential functions, and the corresponding antiderivative rules. Know how to use logarithmic differentiation. Know how to use implicit differentiation.

g) And know the corresponding antiderivative rules.

h) Graphing (all aspects) including asymptotes (make sure to verify asymptotes by computing appropriate limits). Reconstructing graphs from the graph of $f'(x)$ or the number lines for $f'(x)$ and $f''(x)$.

i) Initial value and motion problems including problems with constant acceleration (gravity) using antiderivatives. Motion problems using derivatives.

j) Related rate problems.

k) Max-min problems (with justification).

l) The list above is probably 95% of the material on the final. But there may be something I have missed. See the previous exams and labs. Note: There is at least one problem that asks you to extend your knowledge.

Classwork Day 42: More Motion

1. In the final sprint of a crew race, the challenger is rowing at a constant velocity of 12m/s. At the moment when the leader is 100m from the finish and the challenger is 15m behind, the leader is rowing at 8m/s but is accelerating at 0.5m/s/s. Who wins?

Math 130: Lab XV, Practice on Recent Material

☞ There are **many more practice problems online**. Some answers on the back. Others on line.

1. What are the antiderivatives of these functions? For some you will need to simplify the function first. **Always include** $+c$**.**

a) $\displaystyle\int 2x^{-3/5} + \frac{2}{\sqrt[3]{x^7}}\,dx$ **b)** $\displaystyle\int \frac{x+2}{\sqrt{x}}\,dx$ **c)** $\displaystyle\int \frac{6}{\sqrt{1-t^2}}\,dt$ **d)** $\displaystyle\int \frac{6}{\sqrt{9-t^2}}\,dt$

2. Try these:

a) $\displaystyle\int x(2+x^3)\,dx$ **b)** $\displaystyle\int 6(8^x \ln 8)\,dx$ **c)** $\displaystyle\int \frac{12}{25+x^2}\,dx$ **d)** $\displaystyle\int 4\sin(9x)\,dx$

e) $\displaystyle\int 4e^{-8t}\,dt$ **f)** $\displaystyle\int \sec^2(x/4)\,dx$ **g)** $\displaystyle\int \frac{\sec(-7x)\tan(-7x)}{2}\,dx$ **h)** $\displaystyle\int \frac{6}{t}\,dt$

i) $\displaystyle\int -\frac{1}{5x}\,dx$

3. a) Find the **derivative** of $f(x) = x\ln x - x$.

b) Determine $\int \ln x\,dx$. Hint: Look up.

☞ In the next three problems, we use the fact that the derivative is a rate of change. The rate of change in position is velocity. The rate of change in velocity is acceleration. The rate of change in volume is the flow rate.

4. On Mars the acceleration due to gravity equals 0.39 that of earth or -12.5 ft/sec/sec. Suppose that a Martian calculus student throws his calculus text *upward* at 25 ft/sec off the roof of a building.

a) If it takes 6 seconds to hit the ground, how high is the building?

b) What was the velocity of the book at the instant it the ground?

5. A stone dropped off a cliff hits the ground with speed of 120 ft/s. What was the height of the cliff? (Remember, acceleration is constant at -32ft/s^2.)

6. My Honda Accord accelerates from 0 to 88 ft/sec (60 mph) in 13 seconds.

a) Assume that the acceleration is constant, a. Find the particular velocity function of the car.

b) Find the position function of the car.

c) How far does it travel in this 13 second period?

7. From Lab 14: France has been in the vanguard of high-speed passenger rail travel since the 1970s, and now has a modern rail network capable of accommodating trains running at speeds in excess of 84 m/s (about 300 km/h). Suppose such a train is approaching a station at 84 m/s and begins braking (say at time $t = 0$) at a constant rate of 2.8 m/s^2. How far (in meters) from the railway station did it begin to brake if it stopped right at the station platform? Use the steps below.

a) First determine the velocity function $v(t)$

b) Next, determine how long it takes the train to stop.

c) How far (in meters) from the railway station did it begin to brake if it stopped right at the station platform? (You will need the position function.)

8. Evaluate these limits. ($+\infty$ or $-\infty$ are possible answers).

a) $\displaystyle\lim_{x\to 2} \frac{x^2+x-6}{x^2-3x+2}$ **b)** $\displaystyle\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-3}$ **c)** $\displaystyle\lim_{x\to 2^+} \frac{x^2+1}{2x-x^2}$

d) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x}\right)^{x^2}$ **e)** $\displaystyle\lim_{x\to 0} \frac{\sin^2 9x}{x^2}$ **f)** $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{16x^2-9x}}{3x-7}$

Math 130 Lab XV: Short Answers

More complete answers on line.

1. Recall that $F(x)$ is an **antiderivative** of $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f.

a) $5x^{2/5} - \dfrac{3x^{-4/3}}{2} + c$ **b)** $\dfrac{2x^{3/2}}{3} + 4x^{1/2} + c$ **c)** $6 \arcsin t + c$ **d)** $6 \arcsin \dfrac{t}{3} + c$

2. Be careful

a) $x^2 + \dfrac{1}{5}x^5 + c$ **b)** $6 \cdot 8^x$ **c)** $\dfrac{12}{5} \arctan \dfrac{x}{5} + c$

d) $-\dfrac{4}{9} \cos(9x) + c$ **e)** $-\dfrac{1}{2}e^{-8t} + c$ **f)** $4 \tan \dfrac{x}{4} + c$

g) $-\dfrac{\sec(-7x)}{14} + c$ **h)** $6 \ln |t| + c$ **i)** $-\dfrac{1}{5} \displaystyle\int \dfrac{1}{x}\, dx = -\dfrac{1}{5} \ln |x| + c$

j) $\ln |t + 1| + c$ **k)** $\dfrac{1}{3} \ln |3t + 1| + c$

3. a) $y' = \ln x$. **b)** Looking at part (a): $\int \ln x \, dx = x \ln x - x + c$.

4. a) $s(0) = 75$ ft is the building height.
b) $v(6) = -50$ ft/s.

5. $s(t) = -16t^2 + 225$, so $s(0) = 225$. The cliff height is 225 feet.

6. a) $v(t) = \frac{88}{13}t$ ft/s
b) $s(t) = \frac{44}{13}t^2$ ft/s
c) ft

7. a) We have: constant acceleration $= -2.\ \text{m/s}^2$; $v_0 = 84$ m/s; $s_0 = 0$ m. So $v(t) = at + v_0 = -2.8t + 84$.
b) The train stops when $v(t) = -2.8t + 84 = 0$. Solving gives $t = 84/1.4 = 30$ s
c) $s(t) = \frac{1}{2}at^2 + v_0 t + s_0 = -1.4t^2 + 84t$. So the train travels $s(30) = -1.4(30)^2 + 84(30) = 1260$ m.

8. a) $\frac{0}{0}$ form. Ans: 5
b) $\frac{0}{0}$ form. Ans: 6
c) $-\infty$
d) ∞^0 form. Ans: 1.
e) $\frac{0}{0}$ form. Ans: 81.
f) $\frac{\infty}{-\infty}$ form. Ans: $-\frac{4}{3}$