## Math 130: Not the Final Exam

0. Let $F(x)=\int 6 x^{-1} d x$. Which of these functions, if any, is $F(x)$ ? Careful! How do you check that a function is an antiderivative of another?
a) $\ln x^{6}+c$
b) $6 \ln |6 x|+c$
c) $3 \ln x^{2}+c$
d) $6 x^{0}+c$
e) $6 \ln |x|+c$

## Limits

We have seen that limits are a fundamental builing block for much of the work we did in calculus this term.

1. Evaluate each of these limits using algebraic or calculus techniques.
a) $\lim _{t \rightarrow 3} \frac{t^{2}-4 t+3}{t-3}$
b) $\lim _{x \rightarrow 1^{-}} \frac{x^{2}+2}{x-1}$
c) $\lim _{x \rightarrow 0} \frac{3 \sin x}{\arctan x}$
d) $\lim _{x \rightarrow+\infty} \frac{9 x^{2}+1}{e^{x}}$
e) $\lim _{x \rightarrow 0^{+}}(1+x)^{2 / x}$
f) $\lim _{x \rightarrow-\infty} \frac{\sqrt{9 x^{2}+1}}{4 x+2}$
g) $\lim _{x \rightarrow 1} \frac{\sqrt{3 x+1}-2}{2 x-2}$

## Differentiation

In this section of the final, I want you to explore some different ways of thinking about derivatives: graphically, by using the limit definition, and by using formulas that we have developed.
2. Let's start with some of the basic derivative formulas.
a) $D_{x}(\arcsin x)=$
b) $D_{x}(\sqrt[7]{x})=$
c) $D_{x}(4 \sec x)=$
d) If $f(2)=5$ and $f^{\prime}(2)=-1$, evaluate $\left.D_{x}\left(f\left(2 x^{2}\right)\right)\right|_{x=-1}$.
e) If $f(2)=5$ and $f^{\prime}(2)=-1$, and $g$ is the function graphed below, evaluate $\left.D_{x}(f(x) g(x))\right|_{x=2}$.

3. Use the limit definition of the derivative (not a derivative rule!) to find the derivative of $f(x)=x^{2}-2 x$.
4. Assume that the graph below is $f^{\prime}$, the derivative of $f$. Your job is to supply information about the original function $f$.

a) Create a number line for $f^{\prime}$ to determine the intervals where $f$ increases, decreases and where $f$ has relative extrema.
b) By using the slope of $f^{\prime}$, create a number line for $f^{\prime \prime}$ and use it to determine the intervals where $f$ is concave $u$ and concave down and the location of points of inflection.
c) If $f$ passes through the point $(-3,1)$ indicated with a $\bullet$, draw a potential graph of $f$.
5. Now it's time to do a few complicated derivatives. Use the rules that we've developed in class to find the derivatives of:
a) $g(x)=e^{x \cos x}$
b) $f(x)=3 x^{2} \arcsin x$
c) $f(t)=\ln \left(1+4 t^{6}\right)$
d) $f(x)=\frac{3 x^{3}}{x^{2}+2}$ (Simplify your answer)
e) $y=2 \sin \left(6^{\tan x}\right)$
f) $y=\left(1+x^{2}+e^{\arctan x}\right)^{4}$
g) Find $\frac{d y}{d x}$ if $x^{2}+x^{2} y^{2}+y^{3}=9$.

## Properties of Functions

In this section, I want you to think about different properties of functions and how they relate to one another.
6. a) Draw the graph of a function which is continuous on $[0,5]$ and which has an absolute max at $x=2$ and an absolute min at $x=4$ or indicate why this is impossible.
b) Draw the graph of a function which is continuous on $[0,5]$ and which has an absolute max at $x=2$ and has no absolute min or indicate why this is impossible.
c) Draw the graph of a function on $[0,5]$ which has no absolute max or min or indicate why this is impossible.
d) Draw the graph of a function which is differentiable on $[0,5]$ and which has a critical point at $x=2$ which is not a local extreme point.
e) Draw the graph of a function which is continuous on $[0,5]$ for which $f(0)=-2$ and $f(5)=3$ and which is never 0 , or indicate why this is impossible.
f) Draw the graph of a continuous function on $(0,5)$ which has no absolute max or indicate why this is impossible.
7. a) Carefully state the Mean Value Theorem. Then draw and label a figure which illustrates it.
b) Fill in the table below consistent with the information given. For some parts there are many correct answers. "CTNS" stands for "continuous" and "DNE" stands for "does not exist." Then draw a graph which satisfies all of the conditions in the table and which connects to the two endpoints I've plotted.


| $a$ | $\lim _{x \rightarrow a^{-}} f(x)$ | $\lim _{x \rightarrow a^{+}} f(x)$ | $\lim _{x \rightarrow a} f(x)$ | $f(a)$ | CTNS |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 |  |  |  | Yes |
| 2 |  |  | 3 |  | No |
| 3 | 1 |  |  | 1 | No |
| 4 |  |  | 2 | -1 |  |

## Applications

The reason that most of us (are required to) study calculus is because it has a variety of applications. Let's look at a few.
8. An icicle is in the shape of a right circular cone. At a certain point in time the height is 15 cm and is increasing at the rate of $1 \mathrm{~cm} / \mathrm{hr}$; the radius is 2 cm and decreasing at the rate $\frac{1}{10} \mathrm{~cm} / \mathrm{hr}$. Is the volume ( $V=\frac{1}{3} \pi r^{2} h$ ) increasing or decreasing at this instant? At what rate?
9. Four pens (including the dividers) are to be constructed from 300 meters of fencing. An existing rock wall is to be used as one side. Two students in Math 130 come up with different designs. (Justify your answer.)
a) In the first design, the four pens are constructed in a parallel fashion as shown. What is the maximum area that can be enclosed in this design? (Justify your answer.)

b) In the second design design, the four pens are constructed by using a two-by-two grid as shown. What is the maximum area that can be enclosed in this design?

c) Which design is best? Explain.

## Graphing

Much of the work we have done can be used to produce accurate graphs by plotting a very small number of key points.
10. a) Find and classify the relative extrema of $f(x)=x^{4}-4 x^{3}$. Justify your answer.
b) Define what it means for $x=a$ to be a vertical asymptote of a function $g(x)$.
c) Define what it means for $x=c$ to be a critical number of a function $g(x)$.
11. Draw a detailed graph of the function $f(x)=\frac{2-2 x+2 x^{2}}{x^{2}}$. Make sure to include all extrema, inflections, asymptotes, and the correct concavity. You may use (you do not have to show this): $f^{\prime}(x)=\frac{2 x-4}{x^{3}}$ and $f^{\prime \prime}(x)=\frac{12-4 x}{x^{4}}$.

Put in axes and labels where appropriate

## Antidifferentiation

12. What does it mean (what is the definition) for $F(x)$ to be an antiderivative of the function $f(x)$ ?
13. Determine these antiderivatives
a) $\int x^{2}+12 \cos x d x$
b) $\int 4 e^{2 x}-\frac{8}{\sqrt{4-x^{2}}} d x$
c) $\int \frac{6}{\sqrt[5]{x^{4}}} d x$
d) $\int \frac{s^{3}+2 s+9}{s^{1 / 2}} d s$
e) $\int 3^{x}-\sec ^{2}(8 x) d x$
14. A toy rocket is shot upwards from the edge of a canyon (height 180 ft ) and hit the canyon floor after 4 seconds. What was the initial velocity of the toy rocket? Remember, $a(t)$ is constant here and $a(t)=-32 \mathrm{ft} / \mathrm{s}^{2}$.

## Some Answers

0. a, b, c, and e all are.
1. $2,-\infty, 3,0, e^{2},-\frac{3}{4}, \frac{3}{8}$.
2. $\frac{1}{\sqrt{1-x^{2}}}, \frac{1}{7} x^{-6 / 7}, 4 \sec x \tan x, 4, \approx-6.5$
3. Inc: $(-\infty,-1) \cup(5, \infty)$; Dec: $(-1,5) ; \operatorname{RMax} x=-1 ; \operatorname{RMin} x=5 ; \operatorname{CDn}(-\infty, 3)$, CUp $(3, \infty) ;$ POI $x=3$.
4. $\left.e^{x \cos x}(\cos x-x \sin x) ; 6 \arcsin x+\frac{3 x^{2}}{\sqrt{1-x^{2}}} ; \frac{24 t^{5}}{1+4 t^{6}} ; \frac{3 x^{4}+18 x^{2}}{(x+2)^{2}} ; 2 \cos \left(6^{\tan x}\right) 6^{\tan x} \ln 6 \sec ^{2} x\right) ; 4\left(1+x^{2}+e^{\arctan x}\right)^{3}\left(2 x+\frac{e^{\arctan x}}{1+x^{2}}\right)$; $\frac{-2 x-2 x y^{2}}{2 x^{2} y+3 y^{2}}$.
5. (b) and (e) are impossible.
6. $-2 \pi / 3 \mathrm{~cm}^{3} / \mathrm{hr}$
7. (a) 4500 (b) 3750 (c) (a) is better.
8. a) Crit pts: $x=0$ (neither rmax nor rmin) and $x=3$ RMin
9. $\frac{x^{3}}{3}+12 \sin x+c ; 2 e^{2 x}-8 \arcsin \frac{x}{2}+c ; 30 x^{1 / 5}+c ; \frac{2}{7} s^{7 / 2}+\frac{4}{3} s^{3 / 2}+18 s^{1 / 2}+c ; \frac{3^{x}}{\ln 3}-\frac{\tan 8 x}{8}+c$.
10. $v(0)=19 \mathrm{ft} / \mathrm{s}$
