

Math 130 Practice Final

0. $F(x) = \int 6x^5 dx$... we can check that the answer is correct by taking the derivative of each

a) $\frac{d}{dx} (\ln x^6 + C) = \frac{1}{x^6} \cdot 6x^5 = \frac{6}{x}$ ✓

d) $\frac{d}{dx} (6x^0 + C) = \frac{d}{dx} (6+C) = 0$

b) $\frac{d}{dx} (6 \ln(6x+1) + C) = 6 \cdot \frac{1}{6x} \cdot 6 = \frac{6}{x}$ ✓

e) $\frac{d}{dx} 6 \ln 1 \times 1 = \frac{6}{x}$ ✓

c) $\frac{d}{dx} (3 \ln x^2 + C) = 3 \cdot \frac{1}{x^2} \cdot 2x = \frac{6}{x}$ ✓

so a, b, c & e are correct.

1 a) $\lim_{t \rightarrow 3} \frac{t^2 - 4t + 3 \rightarrow 0}{t - 3 \rightarrow 0} = \lim_{t \rightarrow 3} \frac{(t-3)(t-1)}{t-3} = \boxed{2}$

b) $\lim_{x \rightarrow 1^-} \frac{x^2 + 2 \rightarrow 3}{x-1 \rightarrow 0^-} = \boxed{-\infty}$

c) $\lim_{x \rightarrow 0} \frac{3 \sin x \rightarrow 0}{\arctan x \rightarrow 0} = \lim_{x \rightarrow 0} \frac{3 \cos x}{\frac{1}{1+x^2}} = \frac{3}{1} = \boxed{3}$

d) $\lim_{x \rightarrow \infty} \frac{9x^2 + 1 \rightarrow \infty}{e^x \rightarrow \infty} \stackrel{\text{LH}}{=} \lim_{x \rightarrow \infty} \frac{18x \rightarrow \infty}{e^x \rightarrow \infty} \stackrel{\text{LH}}{=} \lim_{x \rightarrow \infty} \frac{18}{e^x \rightarrow \infty} = 0$

e) $\lim_{x \rightarrow 0^+} (1+x)^{2/x} = y$ cont, switch
 $\ln y = \ln \lim_{x \rightarrow 0^+} (1+x)^{2/x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0^+} \ln(1+x)^{2/x} = \lim_{x \rightarrow 0^+} \frac{2 \ln(1+x)}{x \rightarrow 0^+}$
 $= \lim_{x \rightarrow 0^+} \frac{\frac{2}{1+x}}{1} = 2; \quad \ln y = 2, \text{ so } \lim_{x \rightarrow 0^+} (1+x)^{2/x} = e^2$

f) $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 + 1} \rightarrow \infty}{4x \cdot 2 \rightarrow -\infty} \stackrel{\text{H.P.}}{=} \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2}}{4x} = \lim_{x \rightarrow -\infty} \frac{|3x|}{4x} \stackrel{x < 0}{=} \lim_{x \rightarrow -\infty} \frac{-3x}{4x} = -\frac{3}{4}$

g) $\lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - 2 \rightarrow 0}{2x-2 \rightarrow 0} = \lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - 2}{2x-2} \cdot \frac{\sqrt{3x+1} + 2}{\sqrt{3x+1} + 2} = \lim_{x \rightarrow 1} \frac{(3x-3)}{(2x-2)\sqrt{3x+1} + 2}$
 $= \lim_{x \rightarrow 1} \frac{3(x-1)}{2(x-1)(\sqrt{3x+1} + 2)} = \frac{3}{2(4)} = \frac{3}{8}$ or use L'H

2 a) $D_x(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ (b) $D_x(x^{1/7}) = \frac{1}{7}x^{-6/7}$

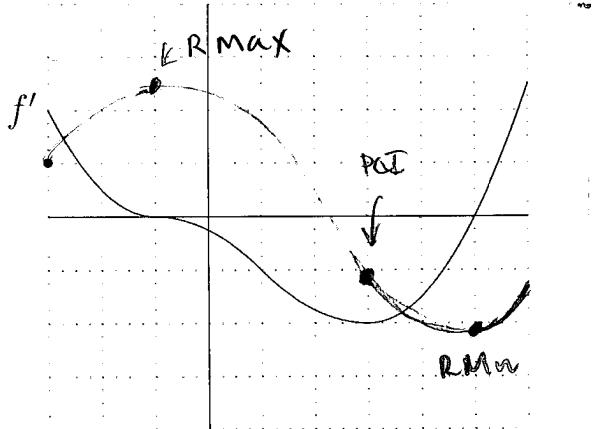
c) $D_x(4 \sec x) = 4 \sec x \tan x$ chain rule

d) $D_x(f(2x^2)) \Big|_{x=-1} = f'(2x^2) \cdot 4x \Big|_{x=-1} = f'(2(-1)^2) \cdot 4(-1) = f'(2) \cdot (-4)$
 $= (-1)(-4) = \boxed{4}$

e) $D_x(f(x)g(x)) \Big|_{x=2} = f'(x)g(x) + f(x)g'(x) \Big|_{x=2} = f'(2)g(2) + f(2)g'(2)$
 $\approx (-1)(1.5) + (5)(-1)$
 ≈ -6.5 slope of g at (2)

$$\begin{aligned}
 3) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h} \\
 &= \lim_{h \rightarrow 0} 2x + h - 2 = \boxed{2x - 2}
 \end{aligned}$$

4



	$+$	0	$R\text{Max}$	$-$	$-$	0	$R\text{Min}$	$+$	$+$
f'	$+$	0	-1	$-$	$-$	0	5	$+$	$+$
f''	$-$	0	$-$	$-$	$-$	0	3	$+$	$+$

Increasing $(-\infty, -1) \cup (5, \infty)$

Decreasing $(-1, 5)$

$R\text{Max}$ @ $x = -1$, $R\text{Min}$ @ $x = 5$

Concave Up $(3, \infty)$

Concave Down $(-\infty, 3)$

or $(-\infty, -1) \cup (-1, 3)$

$P\text{OI}$ @ $x = 3$

du/dx

$$\#5a) D_x(e^{x \cos x}) = e^{x \cos x} (\cos x - x \sin x)$$

$$b) D_x(3x^2 \arcsin x) = 6x \arcsin x + \frac{3x^2}{\sqrt{1-x^2}}$$

$$c) D_t(\ln(1+4t^6)) = \frac{24t^5}{1+4t^6}$$

$$d) \frac{d}{dx}\left(\frac{3x^3}{x^2+2}\right) = \frac{9x^2(x^2+2) - 3x^3(2x)}{(x^2+2)^2} = \frac{3x^4 + 18x^2}{(x^2+2)^2}$$

$$e) \frac{d}{dx}(2 \sin(6^{\tan x})) = 2 \cos(6^{\tan x}) \cdot 6^{\tan x} \ln 6 \cdot \sec^2 x$$

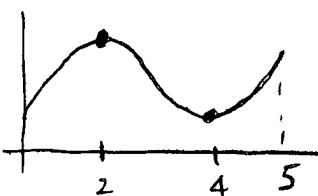
$$f) \frac{d}{dx}(1+x^2 + e^{\arctan x})^4 = 4(1+x^2 + e^{\arctan x})^3 \left(2x + \frac{e^{\arctan x}}{1+x^2}\right)$$

$$g) \frac{d}{dx}(x^2 + x^2y^2 + y^3) = \frac{d}{dx}(1) \Rightarrow 2x + 2xy^2 + 2x^2y \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$

$$(2x^2y + 3y^2) \frac{dy}{dx} = -2x - 2xy^2$$

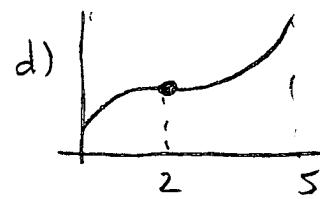
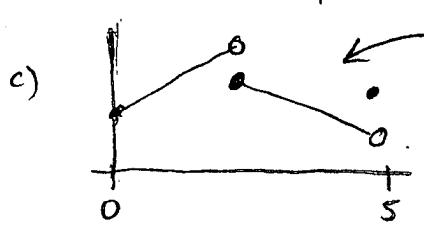
$$\frac{dy}{dx} = \frac{-2x - 2xy^2}{2x^2y + 3y^2}$$

#6 a)

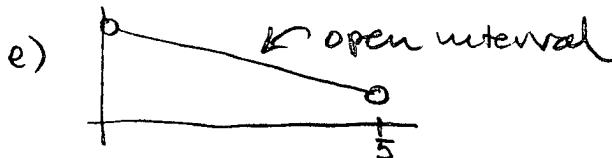


b) Impossible. By Closed Interval Thm must have also max & min on closed interval $[0, 5]$. (Or use EVT)

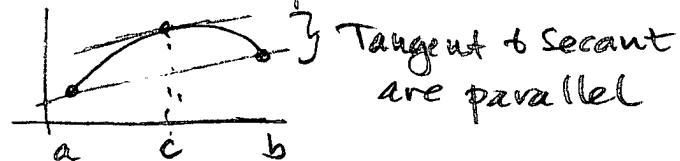
Not cont.



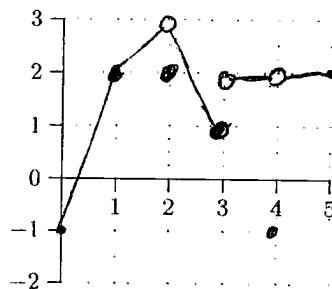
c) Impossible by IVT (Intermediate Value Thm)



- 7a) Assume f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . Then there's a point c in (a, b) so that $f'(c) = \frac{f(b) - f(a)}{b - a}$



- b) Fill in the table below consistent with the information given. For some parts there are many correct answers. "CTNS" stands for "continuous" and "DNE" stands for "does not exist." Then draw a graph which satisfies all of the conditions in the table and which connects to the two endpoints I've plotted.



a	$\lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow a^+} f(x)$	$\lim_{x \rightarrow a} f(x)$	$f(a)$	CTNS
1	2	2	2	2	Yes
2	3	3	3	Not 3	No
3	1	Not 1	DNE	1	No
4	2	2	2	-1	No

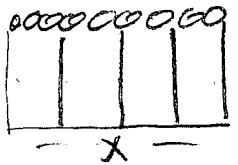
Given $\frac{dh}{dt} \Big|_{\substack{h=15 \\ r=2}} = 1 \text{ cm/hr}$ and $\frac{dr}{dt} \Big|_{\substack{h=15 \\ r=2}} = -\frac{1}{10} \text{ cm/hr}$, Find $\frac{dV}{dt} \Big|_{\substack{h=15 \\ r=2}}$

Relation: $V = \frac{1}{3}\pi r^2 h$

$$\text{Rate: } \frac{dV}{dt} = \frac{1}{3}\pi 2r \frac{dr}{dt} \cdot h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} \Big|_{\substack{h=15 \\ r=2}} = \frac{1}{3}\pi(4) \cdot \left(-\frac{1}{10}\right)(15) + \frac{1}{3}\pi(4)^2(1) = -2\pi + \frac{4}{3}\pi = -\frac{2}{3}\pi \text{ cm}^3/\text{hr}$$

#9) a) Max Area: $A = xy$



$$\text{constraint } x + 5y = 300;$$

$$\text{Eliminate } x = 300 - 5y \quad 0 \leq y \leq 60$$

$$\text{Substitute: } A = (300 - 5y)y = 300y - 5y^2 \quad [0, 60]$$

$$A' = 300 - 10y = 0; \quad y = 30$$

use C.I.T. Check end pts, crit pts: $A(0) = 0$

$$A(60) = 0$$

$$A(30) = 150(30) = 4500 \quad \begin{matrix} \text{Abs} \\ \searrow \\ \text{Max} \end{matrix}$$

b) $A = xy$

$$\text{constraint } 2x + 3y = 300$$

$$\text{Eliminate: } 2x = 300 - 3y; \quad x = 150 - \frac{3}{2}y \quad 0 \leq y \leq 100$$

$$\text{Subst: } A = (150 - \frac{3}{2}y)y = 150y - \frac{3}{2}y^2 \quad [0, 100]$$

$$A' = 150 - 3y = 0, \quad y = 50$$

use C.I.T. Check End pts, crit pts

$$A(0) = 0$$

$$A(100) = 0$$

$$A(50) = (75)(50) = 3750$$

(a) is a better design (more area)

#10 a) $f(x) = x^4 - 4x^3$, $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0, \quad x=0, 3$

$$f' \begin{array}{c} -- 0 -- 0 ++ \\ \hline 0 \quad 3 \end{array} \quad \text{crit pts: } x=0 \text{ Neither Rel Max/Min} \\ \quad \quad \quad x=3 \text{ Rel Min}$$

b) $x=a$ is a vertical asymptote of $g(x)$ if $\lim_{x \rightarrow a^+} g(x) = \pm \infty$

$$\text{or } \lim_{x \rightarrow a^-} g(x) = \pm \infty$$

c) $x=c$ is a critical point of $g(x)$ if either $g'(c) = 0$
or $g'(c)$ DNE

#11 $f(x) = \frac{2-2x+2x^2}{x^2}$ not defined @ $x=0$

$$\text{HA: } \lim_{x \rightarrow +\infty} \frac{2-2x+2x^2}{x^2} \stackrel{\text{HP}}{=} \lim_{x \rightarrow +\infty} \frac{2x^2}{x^2} = 2; \quad \lim_{x \rightarrow -\infty} \frac{2-2x+2x^2}{x^2} \stackrel{\text{HP}}{=} \lim_{x \rightarrow -\infty} \frac{2x^2}{x^2} = 2$$

$y=2$ HA in both directions

$$\text{VA... check @ } x=0: \lim_{x \rightarrow 0^+} \frac{2-2x+2x^2}{x^2} \underset{x \rightarrow 0^+}{\rightarrow} +\infty; \quad \lim_{x \rightarrow 0^-} \frac{2-2x+2x^2}{x^2} \underset{x \rightarrow 0^-}{\rightarrow} +\infty$$

VA @ $x=0$

$$f'(x) = \frac{2x-4}{x^3} = 0 \text{ at } x=2 \quad \text{DNE: } x=0$$

f' $\begin{matrix} + & + & + \\ \text{INC} & | & \text{DEC} \\ 0 & & 2 \end{matrix}$ $\begin{matrix} 0 & + & + \\ \text{INC} & | & \text{INC} \\ 2 & & 0 \end{matrix}$

$x=2$ Rel Min

$$f''(x) = \frac{12-4x}{x^4} = 0 \text{ at } x=3 \quad \text{DNE: } x=0$$

f'' $\begin{matrix} + & + & + \\ \text{INC} & | & \text{INC} \\ 0 & & 3 \end{matrix}$

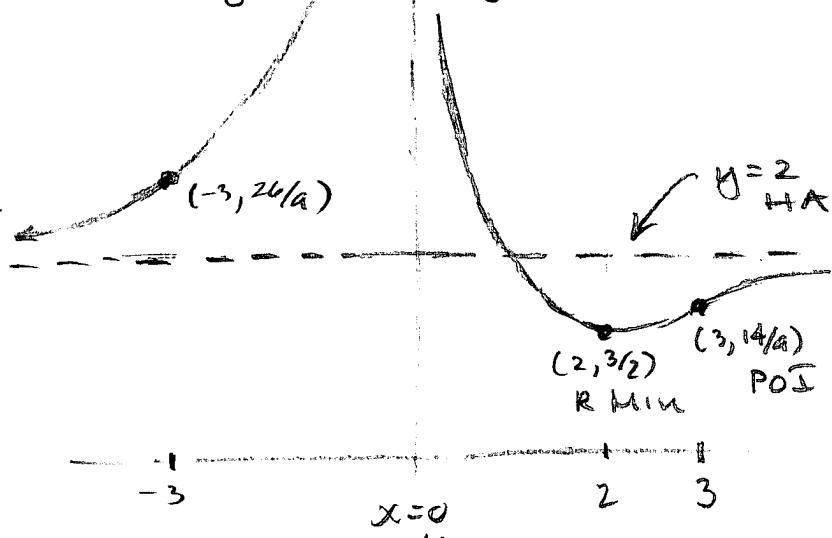
$x=3$ POI

Points: $f(2) = \frac{2-4+8}{4} = \frac{3}{2}$

$$f(3) = \frac{2-6+18}{9} = \frac{14}{9}$$

Additional

$$f(-3) = \frac{2+6+18}{9} = \frac{26}{9}$$



#12

$F(x)$ is an antiderivative of $f(x)$ on I

If $F'(x) = f(x)$ on I

#13 a) $\int x^2 + 12 \cos x \, dx = \frac{x^3}{3} + 12 \sin x + C$ b) $\int 4e^{2x} - \frac{8}{\sqrt{4-x}} \, dx = 2e^{2x} - 8 \arcsin \frac{x}{2} + C$

c) $\int 6x^{-4/5} \, dx = \frac{6x^{1/5}}{1/5} + C = 30x^{1/5} + C$

d) $\int \frac{s^3 + 2s + 9}{s^{1/2}} \, ds = \int s^{5/2} + 2s^{1/2} + 9s^{-1/2} \, ds = \frac{2}{7}s^{7/2} + \frac{4}{3}s^{3/2} + 18s^{1/2} + C$

e) $\int 3^x - \sec^2(8x) \, dx = \frac{3^x}{\ln 3} - \frac{1}{8} \tan(8x) + C$

#14 Given $a(t) = -32 \text{ ft/s}^2$ $s(0) = 180$, $s(4) = 0$

Find $v(0)$

$$v(t) = \int a(t) \, dt = \int -32 \, dt = -32t + C \quad \begin{matrix} \text{We can't evaluate } C \\ \text{no given velocity} \end{matrix}$$

$$s(t) = \int v(t) \, dt = \int -32t + C \, dt = -16t^2 + Ct + d$$

$$s(0) = -0 + 0 + d = 180 \text{ so } d = 180, \quad s(t) = -16t^2 + Ct + 180$$

Now $s(4) = 0 = -16(4^2) + 4C + 180; \quad 4C = -180 + 16^2 = 76$

$$C = 19$$

$$s(t) = -16t^2 + 19t + 180$$

$$v(t) = -32t + 19 \quad \text{so } v(0) = -0 + 19 = \boxed{19 \text{ ft/s}}$$

initial velocity

