Math 130, Lab 11

1. a) Let $f(x) = x + \dfrac{4}{x}$. Determine which critical points are local maxima, minima, and which are not extreme and mark this on a number line for f'.

To do this which theorem did you use? ______________________________

b) Determine which points are inflections and mark the concavity on a number line for f''.

To do this which theorem did you use? ______________________________

c)

Plot and label the critical and inflection points. Indicate coordinates.

$f(x) = x + \frac{4}{x}$ is not defined at $x = 0$.

What happens as $x \to 0^+$? Evaluate $\lim_{x\to 0^+} x + \frac{4}{x}$.

What happens as $x \to 0^-$? Evaluate $\lim_{x\to 0^-} x + \frac{4}{x}$.

Use an appropriate scale and axes so that the 'action'

2. a) Let $f(x) = (x^3 - 8)^{1/3}$. Determine which critical points are local maxima, minima, and which are not extreme and mark this on a number line for f'.

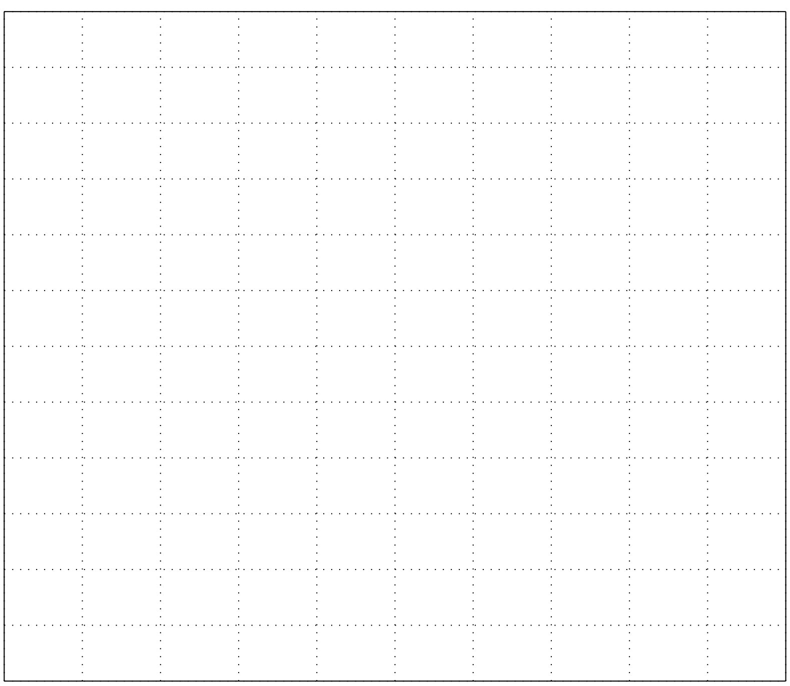
To do this which theorem did you use? ______________________________

b) Determine which points are inflections and mark the concavity on a number line for f''. Hint: To save time use:

$$f''(x) = \frac{-16x}{(x^3 - 8)^{5/3}}$$

To do this which theorem did you use? ______________________________

c)



Plot and label the critical and inflection points.

Indicate coordinates.

Plot and label y (and x) intercepts.

Use appropriate scales and axes so that the 'action' near critical numbers can be seen.

The scales may be different on each axis if necessary.

3. a) $f(x) = 6x^{1/3} - 2x$. Determine which critical points are local maxima, minima, and which are not extreme and mark this on a number line for f'.

To do this which theorem did you use? ____________________________

b) Determine which points are inflections and mark the concavity on a number line for f''.

To do this which theorem did you use? ____________________________

c)

Plot and label the critical and inflection points.

Indicate coordinates.

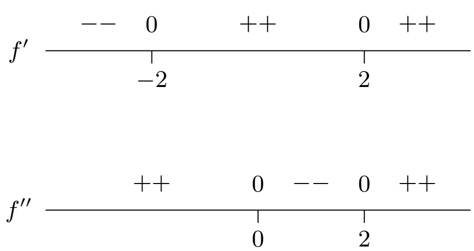
Plot and label y (and x) intercepts.

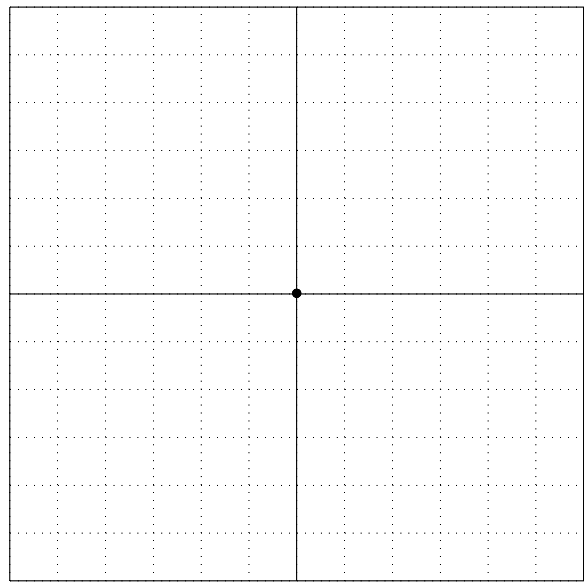
Use appropriate scales and axes so that the 'action' near critical numbers can be seen.

The scales may be different on each axis if necessary.

4. Below I give you information about the derivatives of two *continuous* functions. First determine where each function is increasing and decreasing. Next classify each critical point as a relative max, relative min, or neither. Then determine where the function is concave up and down and the location of any inflections. Then for each, sketch the graph function that would have derivatives like those given. Indicate on your graph which points are local extrema. DNE indicates that the derivative does not exist at the point (though the original function does). You will need to make up function values for the critical points that are consistent with the given information. The only point you know in each case is that the function passes through $(0, 0)$, which is marked on each graph.

a)





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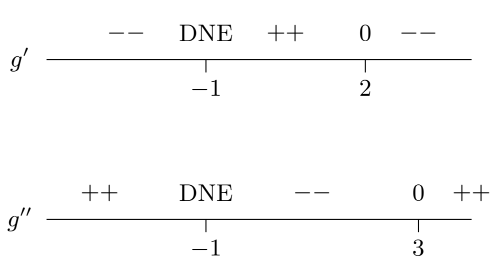
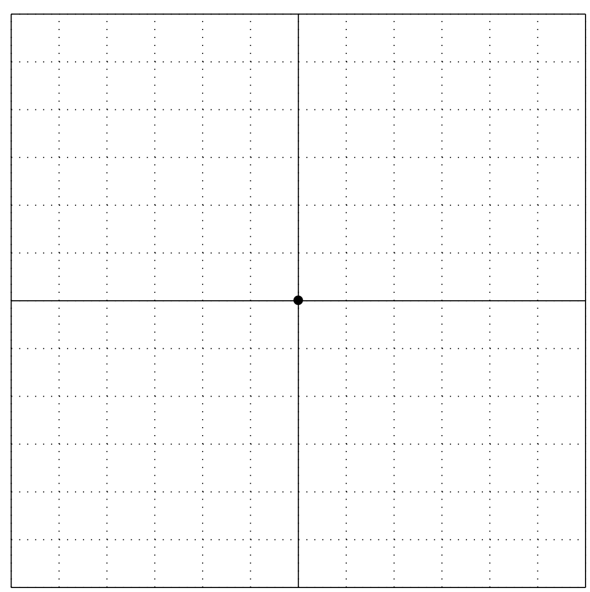
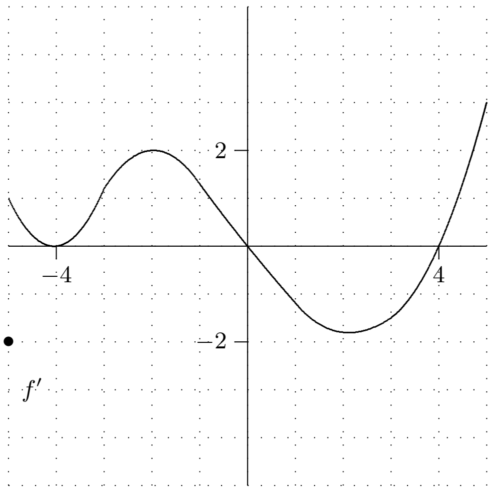




5.



To the left is the graph of $f'(x)$, the **derivative** of $f(x)$.
This is the graph of $f'(x)$, NOT the graph of $f(x)$. You can read off the values of f' for your 'number line' directly from it. E.g., $f'(2) = -1.8$ (NOT 0)

a) Convert the graph of $f'(x)$ into number line information for $f'(x)$.
Use the number line below the graph. Mark where is $f'(x)$ positive, negative, and 0.

b) Use the number line to classify each critical point of f (loc extrema or not).

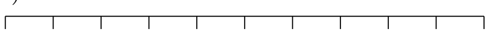
c) Use the number line to find where f is increasing and decreasing.

d) Now create a number line for f''. Remember f'' is the derivative (slope) of f'. So where is the slope of f' positive? Negative? 0?
Use this number line to determine concavity and inflections.

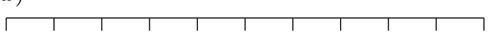
e Sketch the graph of the original function $y = f(x)$. Use the same axes.

You will have to make up values for the function consistent with $f'(x)$.
Start your graph at $(-5, -2)$.

$f'(x)$



$f''(x)$

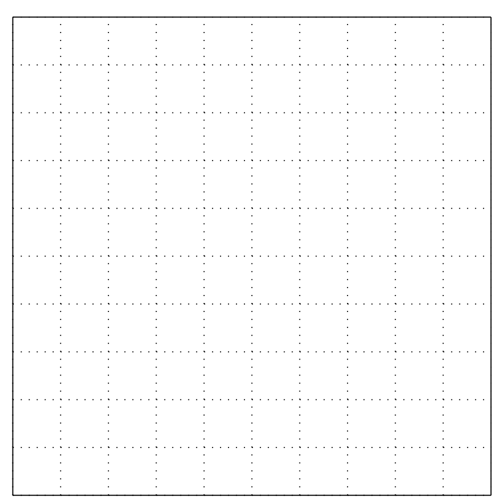


6. This is a combination graphing and optimization problem. A patient takes a 20 mg dose of a drug (in pill form). The drug is absorbed and metabolized by the patient so that after t hours there are $f(t) = 20te^{-t}$ mg in the patients bloodstream, where for the time interval $[0, \infty)$.

a) Find and classify the critical numbers of this function. What theorem did you use?

b) Find the **absolute maximum value** or amount of medication in the bloodstream. At what time does this occur? Justify your answer with a theorem that we discussed yesterday.

c) Sketch a graph of the function on the interval $[0, \infty)$. Include critical points and endpoint. Make sure to 'justify' your graph by showing your work and carefully using the first and second derivatives.



Math 130, Lab 11: Answers

1. a) $f(x) = x + \dfrac{4}{x}$. Notice $x = 0$ is not in the domain of f (so it cannot be a critical number). Critical points:

$$f'(x) = 1 - \frac{4}{x^2}, \text{ DNE at } x = 0$$

$$f'(x) = 1 - \frac{4}{x^2} = 0 \Rightarrow 1 = \frac{4}{x^2} \Rightarrow x^2 = 4$$

So $x = -2,\ 2$ and watch out for $x = 0$ on the number line since f' is not continuous.

		R Max		not in domain		R Min	
	$+++$	0	$---$	$*$	$---$	0	$+++$
f'	Incr	-2	Decr	0	Decr	2	Incr

To do this which theorem did you use? First Derivative Test

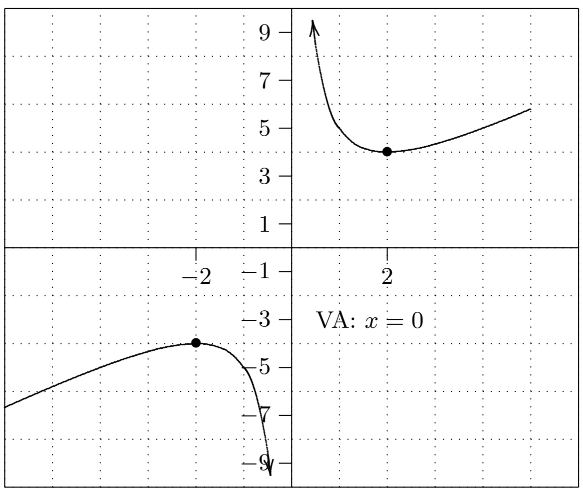
b)

$$f''(x) = \frac{d}{dx}\left(1 - \frac{4}{x^2}\right) = \frac{d}{dx}\left(1 - 4x^{-2}\right) = 8x^{-3} = \frac{8}{x^3} = 0 \text{ Never!} \qquad f'(x) \text{ DNE at } x = 0.$$

	$----$	DNE	$++++$
f''	Con Dn	0	Con Up

To do this which theorem did you use? Concavity Test

c)



At the critical numbers:
$f(-2) = -2 + \frac{4}{-2} = -4$ and $f(2) = 2 + \frac{4}{2} = 4$

There are no intercepts

Finally, $\lim\limits_{x\to 0^+} x + \underbrace{\dfrac{4}{x}}_{0^+} = +\infty$ $x = 0$ VA.

and $\lim\limits_{x\to 0^-} x + \underbrace{\dfrac{4}{x}}_{0^-} = -\infty$ $x = 0$ VA.

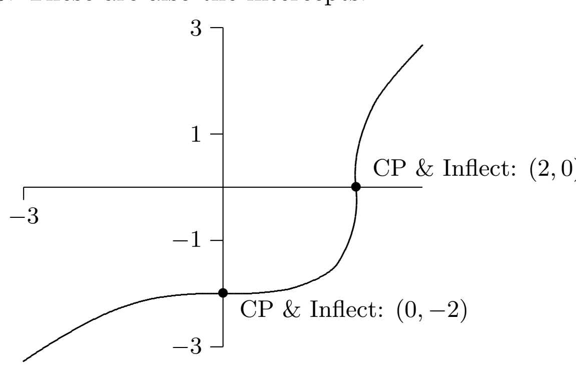
2. a) Critical numbers: $f'(x) = \frac{1}{3}(x^3-8)^{-2/3}3x^2 = \frac{x^2}{(x^3-8)^{2/3}} = 0$ at $x = 0$ and DNE at $x = 2$. (See number line in part (d) below.)

b) Potential inflections: $f''(x) = \frac{-16x}{(x^3-8)^{5/3}} = 0$ at $x = 0$ and DNE at $x = 2$. (See number line in part (d) below.)

c) Evaluate f at critical and inflection points: $f(0) = -2$, $f(2) = 0$. These are also the intercepts.

d)

		Neither		Neither	
	$+++$	0	$++$	$* +$	
f'		0		2	
		INFL		INFL	
	$--$	0	$++$	$* -$	
f''		0		2	



3. a) $f(x) = 6x^{1/3} - 2x$. Critical numbers:

$$f'(x) = 2x^{-2/3} - 2 = \frac{2}{x^{2/3}} - 2 = 0 \Rightarrow 1 = \frac{2}{x^{2/3}} \Rightarrow x^{2/3} = 1 \Rightarrow x = 1,\ -1 \text{ and } f'(x) \text{ DNE at } x = 0.$$

		R Min		CP		R Max		
f'	$---$	0	$++$	DNE	$++$	0	$---$	
	Decr	-1	Incr	0	Incr	1	Decr	

To do this which theorem did you use? First Derivative Test

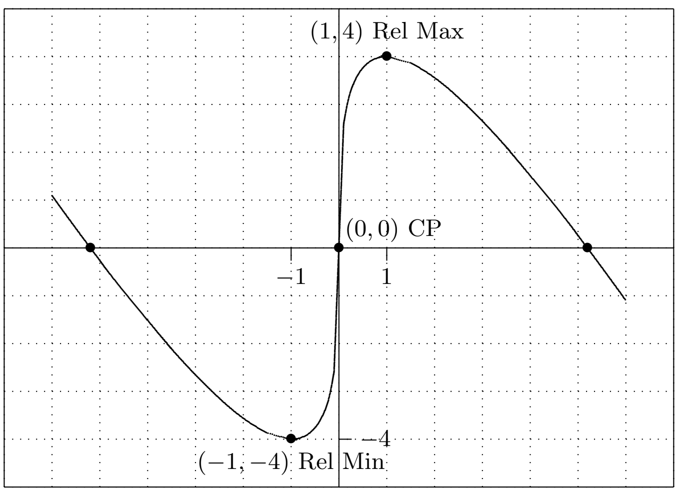
b)

$$f''(x) = -4x^{-5/3} = -\frac{4}{x^{5/3}} = 0 \text{ Never!} \qquad f'(x) \text{ DNE at } x = 0.$$

		Inflect	
f''	$++++$	DNE	$----$
	Con Up	0	Con Dn

To do this which theorem did you use? Concavity Test

c)



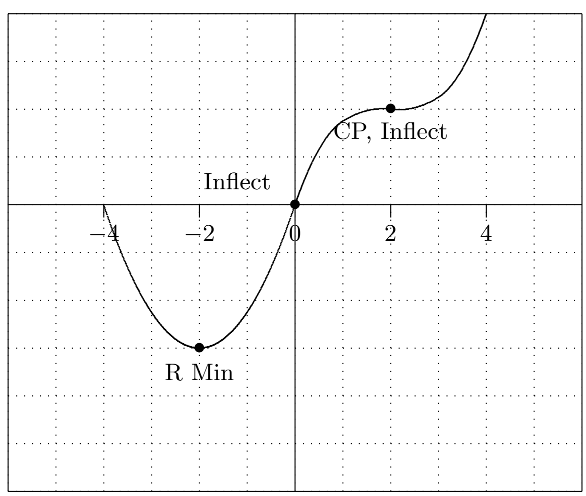
At the critical numbers:
$f(0) = 0$ and
$f(-1) = -4$.
$f(1) = -4$.
x intercept: $6x^{1/3} - 2x = 0 \Rightarrow x = 0$ and
$6x^{1/3} = 2x \Rightarrow 3 = x^{2/3} \Rightarrow x = \pm 3^{3/2} = \pm 5.196$

4. a) There's a local min at $x = -2$. Note: *There is a horizontal tangent at* $x = 2$ but no max or min.

a)

		R Min		CP, Neither	
f'	$--$	0	$++$	0	$++$
	Decr	-2	Incr	2	Decr

		Infl		Infl	
f''	$++$	0	$--$	0	$++$
	Con Up	0	Con Dn	2	Con Up

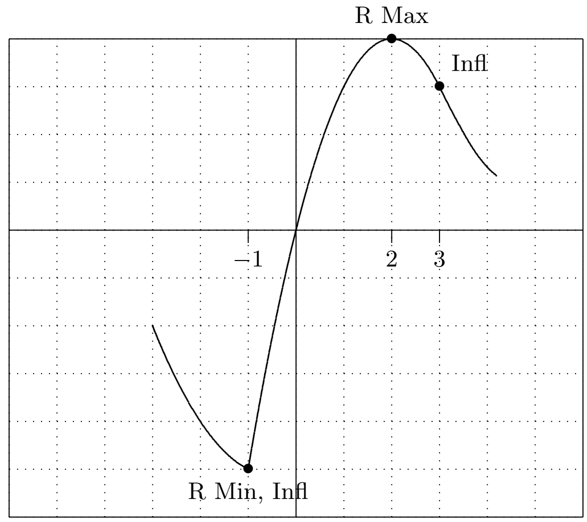


b) There's a local min (and a corner) at $x = -1$ and a local max at $x = 2$.

b)

		R. Min		R. Max	
g'	$--$	DNE	$++$	0	$--$
	Decr	-1	Incr	2	Decr

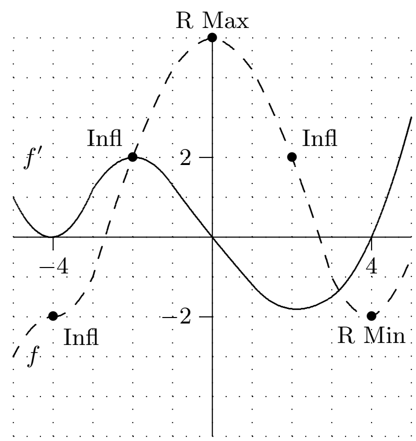
		Infl		Infl	
g''	$++$	DNE	$--$	0	$++$
	Con Up	-1	Con Dn	3	Con Up



5.

		Neither		R Max		R Min	
f'	$+$	0	$++$	0	$----$	0	$++$
	Incr	-4	Incr	0	Decr	4	Incr

		Infl		Infl		Infl	
f''	$--$	0	$++$	0	$----$	0	$+++$
	Con Dn	-4	Con Up	-2	Con Dn	-2	Con Up



6. a) Use the First Derivative Test. $f'(t) = 20[e^{-t} - te^{-t}] = 20e^{-t}(1-t) = 0$ at $t = 1$.

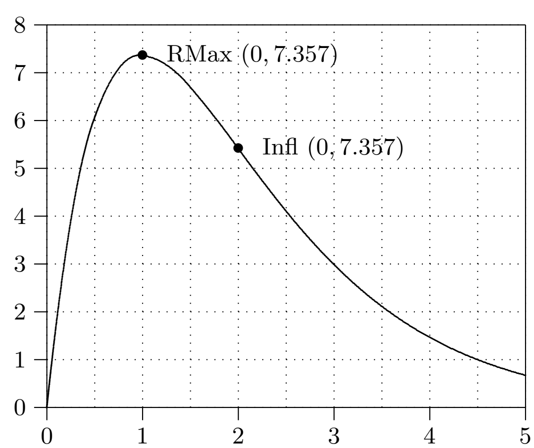
		R Max	
f'	$++$	0	$----$
	[0 Incr	1	Decr

b) We cannot apply CIT because the interval is the wrong type. Nonetheless, the function is differentiable on $[0, \infty)$. On the interval $[0, \infty)$ the function is differentiable and has a single critical point at $t = 1$, where there is a relative max. By the SCPT (Single Critical Point Theorem), if a differentiable function on an interval has a single critical point and it is a relative extreme point, then it is an absolute extreme point. Here, we have a single critical point at $t = 1$, its a relative max, so it must be an absolute max. And the max value is $f(1) = 20e^{-1} \approx 7.358$.

c) The first derivative info is above. $f''(t) = -20[e^{-t}(1-t) + 20e^{-t}(-1) = 20e^{-1}(2-t) = 0$ at $t = 2$.

Evaluate f at critical points and inflections: $f(1) = 20e^{-1} \approx 7.358$ and $f(2) = 40e^{-2} \approx 5.413$.

		POI	
f''	$---$	0	$+++$
	Con Dn	2	Conc Up



Post-Lab 11: Take-Home Quiz. Names: ______________________________

There's a two point bonus if you do this with a partner. **Hand in only one copy**.

1. a) $f(x) = x - 3x^{2/3}$. Determine which critical points are local maxima, minima, and which are not extreme and mark this on a number line for f'.

To do this which theorem did you use? ____________________

b) Determine which points are inflections and mark the concavity on a number line for f''.

To do this which theorem did you use? ____________________

c)

Plot and label the critical and inflection points.
Indicate coordinates.

Plot and label y (and x) intercepts.

Use appropriate scales and axes so that the 'action' near critical numbers can be seen.

The scales may be different on each axis if necessary.