Math 130 Practice Test 1

Some of these questions appear on the optional WeBWorK PracticeTest1. Try them before the answers are posted on line! **Caution**: There may be some topics not covered by this review.

1. a) Carefully define "f is **continuous** at a."

b) Carefully define ""f is **left continuous** at a."

c) Carefully define "f has a **removable discontinuity** at a."

d) Carefully define "f has a **vertical asymptote** at a."

e) Carefully define "f has a **horizontal asymptote**."

f) What is the general formula for the slope of the secant line through the points $(x, f(x))$ and $(a, f(a))$?

g) The position of an object at time x is given by $f(x)$. What is the general formula for the average velocity of the object on the time interval $[a, x]$?

2. a) Give a function (a formula) that is continuous for all x and give a reason why it is continuous.

b) Draw the graph of a function that is neither right nor left continuous at $x = -2$ even though $\lim_{x\to-2} f(x)$ exists.

c) Draw another function that is right but not left continuous at $x = -1$.

d) Draw another function defined on $[-6, -4]$ that is continuous there.

e) Draw, if possible, a function continuous on $(-4, -2)$ but not on $[-4, -2]$.

f) Draw, if possible, a function continuous on $[2, 5]$ but not on $(2, 5)$.

g) Draw an example of a function that has a removable discontinuity at $x = -2$.

h) Draw an example of a function that has a removable discontinuity and a vertical asymptote at $x = 2$ or explain why this is impossible.

i) Draw a continuous function f on the closed interval $[0, 5]$ so that $f(0) = 2$ and $f(5) = 11$ but $f(x)$ is never 7.

3. Evaluate the following limits. They are all mixed up and look very similar; some are infinite, some require more work. **Strategy:** First determine what the numerator and denominator are approaching. Indeterminate form $\frac{0}{0}$ means more work! A "non-zero number over 0" means you need to pay attention to signs and determine whether the limit is $\pm\infty$. Others may require no work at all (justify as "polynomial", etc.). On the test you will need to show all your work to obtain full credit. Similar problems require different approaches.

a) $\displaystyle\lim_{x\to1} \frac{x^2+x-2}{x^2-x+2}$ **b)** $\displaystyle\lim_{x\to3^-} \frac{x^2-9}{x^2+2x-15}$ **c)** $\displaystyle\lim_{x\to5^+} \frac{x^2+5x}{x^2-25}$ **d)** $\displaystyle\lim_{x\to1} \frac{x+1}{2x(x-1)^2}$

e) $\displaystyle\lim_{x\to2} \frac{\frac{1}{x+2}-\frac{1}{4}}{x-2}$ **f)** $\displaystyle\lim_{x\to-2^-} \frac{\frac{1}{x+2}-\frac{1}{4}}{x-2}$ **g)** $\displaystyle\lim_{h\to0} \frac{\frac{2}{x+h}-\frac{2}{x}}{h}$ **h)** $\displaystyle\lim_{x\to4^+} \frac{x-4}{\sqrt{x}-2}$

i) $\displaystyle\lim_{x\to4^-} \frac{x-6}{2-\sqrt{x}}$ **j)** $\displaystyle\lim_{x\to a} \frac{2\sqrt{x}-2\sqrt{a}}{x-a}$ **k)** $\displaystyle\lim_{x\to2^+} \frac{x-1}{\sqrt{x-2}}$ **l)** $\displaystyle\lim_{x\to2} \frac{x^2+x}{x^2-2x}$

m) $\displaystyle\lim_{x\to2^-} \frac{x+3}{x^3-4x^2+4x}$ **n)** $\displaystyle\lim_{x\to-\infty} \frac{2x+1}{\sqrt{25x^2+9x}}$ **o)** $\displaystyle\lim_{x\to-\infty} \frac{x^2-9x^3}{x^2+2x-15}$ **p)** $\displaystyle\lim_{x\to1} \cos\left(\frac{x^2+x-2}{x^2-x+2}\right)$

q) $\displaystyle\lim_{x\to\pi/2} (e^{\sin x})$ **r)** $\displaystyle\lim_{x\to0^+} \frac{e^x+\cos x}{\sin^2 x}$

s) Try $\displaystyle\lim_{x\to1} \frac{2}{x-1} - \frac{2}{x(x-1)}$. Hint: Use a common denominator.

4. Which of the following are polynomials? Rational (and not a polynomial)? Neither?

a) $\dfrac{x^2+x-2}{x^2-x+2}$ **b)** $\sqrt{x^2-x+12}$ **c)** $4x^8-9x^4+3x+\frac{1}{2}$ **d)** $\dfrac{2}{x(x+1)}$

e) $x(2x-5)(x-1)$ **f)** $2x(x^2-x+1)^{-1}$ **g)** $\dfrac{\sin 2x}{\sin x}$ **h)** $\dfrac{x-4}{\sqrt{x}-2}$

5. a) Suppose that $f(x) = \sqrt{x+1}$. Find the general formula slope of the secant line through the points $(3, f(3))$ and $(x, f(x))$.

b) Find the slope of the curve right at $x = 3$ by taking the limit: $\lim_{x\to3}$ Secant Slope.

c) What is the equation of this tangent line? Remember it passes through the point $(3, f(3))$.

6. The position of an object moving along a straight line after x seconds is given by $f(x) = x^2 - 5x + 6$ meters, where x is time.

a) Find the general formula for its average velocity on the time interval from 1 to x.

b) Determine the instantaneous velocity function right at time 1 by evaluating an appropriate limit.

c) Notice that the process in this problem and the previous problem are entirely the same. So fill in the analogy: Secant Slope is to [   ] as Average Velocity is to Instantaneous Velocity.

7. Consider the function defined by $y = f(x) = \begin{cases} \frac{x-3}{x^2-2x-3}, & \text{if } x \neq 3 \\ 2, & \text{if } x = 3 \end{cases}$. Determine whether f is continuous at $x = 3$. Show your work using limits.

8. Consider the piecewise function defined by $y = f(x) = \begin{cases} \frac{x^2+2x-3}{x^2+4x-5}, & \text{if } x < 1 \\ z, & \text{if } x = 1 \\ \frac{\sqrt{x}-1}{1-x}, & \text{if } x > 1 \end{cases}$.

a) What value of z would make f LEFT continuous at $x = 1$.

b) Using this value of z from part (a), is f continuous at $x = 1$?

c) With your given value of z, determine the intervals of continuity for f.

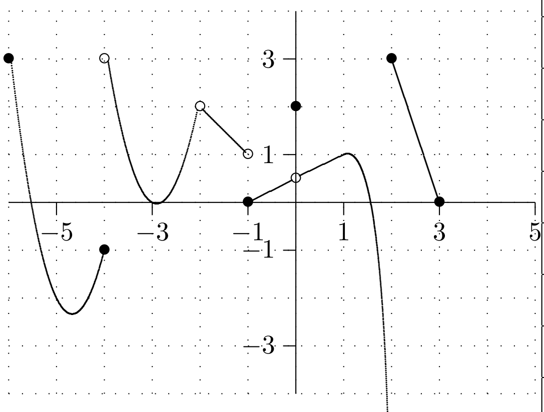
9. Consider the piecewise function defined by $y = f(x) = \begin{cases} \frac{x}{\sqrt{x+4}-2}, & \text{if } x < 0 \\ \frac{3x+4}{x^6+x^2+1}, & \text{if } x \geq 0 \end{cases}$.

a) Determine whether f is continuous at $x = 0$. Show your work using limits.

b) Determine whether f is left or right continuous at $x = 0$. Show your work using limits.

c) Without doing any limit calculations, explain why f is or is not continuous at $x = 4$. (A brief sentence should suffice.)

10. a) Use the graph of f to evaluate each of the expressions in the table or explain why the value does not exist. For the "Continuity" column, your answer should be: "Left" if the function is **continuous from the left only**, "Right" if the function is **continuous from the right only**, "Neither" if it is not continuous, and "Both" if it is continuous (from both sides) and "Removable" if it has a removable discontinuity. Use $+\infty$ or $-\infty$ where appropriate. For the last few points, complete the graph so that the given information is true.



a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Ctns, Rem
-5					
-4					
-2					
-1					
1					
2					
3		2			
4				3	Right only

b) Find a point a in the graph above where the limit exists and $f(a)$ exists, but $f(x)$ is not continuous.

c) Is f continuous on $[2, 3]$? On $[-4, -2]$? On $(-4, -2)$?

11. a) Determine using limits and the definition where $f(x) = \frac{x^2+7x+10}{x^2+4x-5}$ has vertical asymptotes.

b) Does $f(x)$ have any removable discontinuities? If so, demonstrate this using an appropriate limit.

c) Determine the intervals of continuity.

12. Let $f(x) = \begin{cases} \frac{x^2+3x}{x}, & \text{if } x > 0 \\ \frac{x^2-1}{x^2+x}, & \text{if } x < 0 \end{cases}$. Does f have a removable discontinuity at $x = 0$? Explain carefully with reference to the definition of RD.

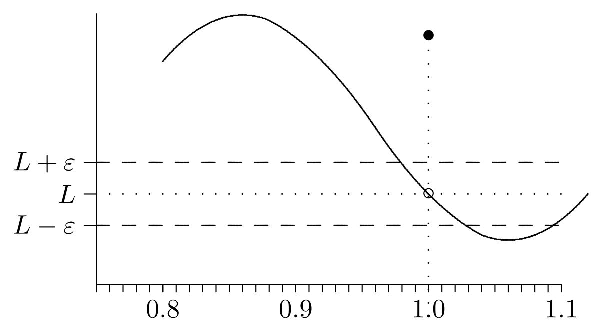
a) Does f have an RD at $x = 0$? Explain using limits and the definition of RD.

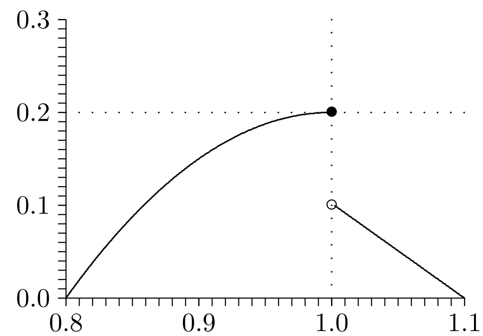
b) Does f have a VA at $x = 0$? Explain using limits and the definition of VA.

13. Fill in the following table using your knowledge of limits and continuity. For some parts, there are many acceptable answers.

a	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$	$\lim\limits_{x\to a} f(x)$	$f(a)$	Left Ctns	Right Ctns	Ctns	RD, VA
-5	2				Yes	No		
-4	4							RD
-3	1	2		2				
-2			-1	4				
-1	4	5			Yes			
0				-2	Yes	Yes		
1	2	2			No			
2			3			No		
4	$+\infty$	4				Right		

14. a) In the figure below (left), for the given choice of ε, find and draw a δ interval about $a = 1$ which satisfies the limit definition Note scale!





b) In the function on the right, $f(1) = .2$. However, show that $\lim_{x\to 1} f(x) \neq 0.2$ by finding an $\varepsilon > 0$ (draw the horizontal band) for which no corresponding δ can be found. Explain why your ε works.

c) Is either of the functions above continuous at $a = 1$? Why?

d) Does either have a removable discontinuity at $a = 1$? Why?

15. a) Use the formal definition of limit to show that: $\lim\limits_{x\to 1} 4x + 6 = 10$. Be careful.

b) To make $f(x)$ within $\varepsilon = 0.001$ of the limit $L = 2$ how would you choose δ?

16. Fill in the exact values (no decimals).

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					

17. Consider the piecewise function defined by $f(x) = \begin{cases} \frac{x^2+x}{4x}, & \text{if } x < 0 \\ 1, & \text{if } x = 0 \\ \frac{\frac{1}{2} - \frac{1}{x+2}}{x}, & \text{if } x > 0 \end{cases}$.

a) Is f continuous from the left at $x = 0$? Right? Is f continuous at $x = 0$. Justify your answer with limits.

b) Does f have an RD at $x = 0$? Explain using limits and the definition of RD.

c) Does f have a VA at $x = 0$? Explain using limits and the definition of VA.

d) Give the intervals of continuity of f.

18. In one of the questions below, the Intermediate Value Theorem can be used to guarantee that the equation has a solution. In the other it cannot. Determine which is which and carefully explain why a solution must exist in one case but not the other.

a) Can the Intermediate Value Theorem be used to guarantee that $f(x) = 7x^4 - 4x^3 - 2x^2 + x = 3$ in the interval $[0, 1]$?

b) Can the Intermediate Value Theorem be used to demonstrate that $f(x) = e^x - 3x^2 = 0$ in the interval $[0, 3]$?

Math 130 Practest 1 Answers

1. a) f is **continuous** at a if (1) $f(a)$ exists, (2) $\lim_{x\to a} f(x)$ exists, and (3) $\lim_{x\to a} f(x) = f(a)$.

b) $f(x)$ is **continuous from the left at** $x = a$ if $\lim_{x\to a^-} f(x) = f(a)$.

c) f has a **removable discontinuity** at a if (1) $\lim_{x\to a} f(x)$ exists and (2) $\lim_{x\to a} f(x) \neq f(a)$.

d) f has a **vertical asymptote** at $x = a$ if either $\lim_{x\to a^+} f(x) = +\infty$ or $-\infty$ or $\lim_{x\to a^-} f(x) = +\infty$ or $-\infty$.

e) The line $y = L$ is a **horizontal asymptote** (HA) for the graph of $f(x)$ if either $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

f) $\dfrac{f(x) - f(a)}{x - a}$

g) The same as above.

2. a) Any polynomial, e.g., $p(x) = x^2 + 1$, is continuous for all x

b) See the graph in Problem 10 at $x = -2$, for example.

c) See the graph in Problem 10 at $x = -1$.

d) See the graph in Problem 10 on $[-6, -4]$.

e) See the graph in Problem 10 on $[-4, -2]$.

f) Impossible. By definition, continuous on closed interval $[a, b]$ includes being continuous on the open interval (a, b).

g) See the graph in Problem 10 at $x = -2$.

h) Impossible. If there is a removable discontinuity the limit exists at 2. If there is a vertical asymptote the limit does not exist (becomes $\pm\infty$).

i) Impossible by the Intermediate Value Theorem.

3. a) $\displaystyle\lim_{x\to1} \frac{x^2+x-2}{x^2-x+2} = \frac{0}{2} = 0$. (Rational. Just evaluate since denominator is not 0.)

b) $\displaystyle\lim_{x\to3^-} \frac{x^2-9}{x^2+2x-15} = \lim_{x\to3^-} \frac{(x-3)(x+3)}{(x-3)(x+5)} = \lim_{x\to3^-} \frac{x+3}{x+5} = \frac{6}{8} = \frac{3}{4}$

c) $\displaystyle\lim_{x\to5^+} \frac{x^2+5x}{x^2-25} = \lim_{x\to5^+} \frac{x(x+5)}{(x-5)(x+5)} = \lim_{x\to5^+} \frac{x}{x-5} \to \frac{5}{0^+} = +\infty$

d) $\displaystyle\lim_{x\to1} \frac{x+1}{2x(x-1)^2} \to \frac{2}{(2)(0^+)^2} = +\infty$

e) $\displaystyle\lim_{x\to2} \frac{\frac{1}{x+2} - \frac{1}{4}}{x-2} = \lim_{x\to2} \frac{\frac{4-(x+2)}{4(x+2)}}{x-2} = \lim_{x\to2} \frac{2-x}{4(x+2)(x-2)} = \lim_{x\to2} \frac{-1}{4(x+2)} = -\frac{1}{16}$

f) As above $\displaystyle\lim_{x\to-2^-} \frac{\frac{1}{x+2} - \frac{1}{4}}{x-2} = \lim_{x\to-2^-} \frac{2-x}{4(x+2)(x-2)} = \lim_{x\to-2^-} \frac{-1}{4(x+2)} \to \frac{-1}{4(0^-)} = +\infty$

g) $\displaystyle\lim_{h\to0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to0} \frac{\frac{2x-2(x+h)}{x(x+h)}}{h} = \lim_{h\to0} \frac{-2h}{hx(x+h)} = \lim_{h\to0} \frac{-2}{x(x+h)} = -\frac{2}{x^2}$

h) $\displaystyle\lim_{x\to4^+} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to4^+} \frac{(x-4)(\sqrt{x}+2)}{x-4} = \lim_{x\to4^+} \sqrt{x} + 2 = 4.$

i) $\displaystyle\lim_{x\to4^-} \frac{x-6}{2-\sqrt{x}} \to \frac{-2}{0^+} \to -\infty$

j) $\displaystyle\lim_{x\to a} \frac{2\sqrt{x}-2\sqrt{a}}{x-a} \cdot \frac{2\sqrt{x}+2\sqrt{a}}{2\sqrt{x}+2\sqrt{a}} = \lim_{x\to a} \frac{4x-4a}{(x-a)(2\sqrt{x}+2\sqrt{a})} = \lim_{x\to a} \frac{4}{2\sqrt{x}+2\sqrt{a}} = \frac{4}{4\sqrt{a}} = \frac{1}{\sqrt{a}}$

k) $\displaystyle\lim_{x\to2^+} \frac{x-1}{\sqrt{x-2}} \to \frac{1}{0^+} = +\infty$

l) $\displaystyle\lim_{x\to2} \frac{x^2+x}{x^2-2x} = \lim_{x\to2} \frac{x(x+1)}{x(x-2)} = \lim_{x\to2} \frac{x+1}{x-2}$. The denominator goes to 0, but the numerator does not. Use one-sided limits.

$$\lim_{x\to2^+} \frac{x^2+x}{x^2-2x} = \lim_{x\to2^+} \frac{x+1}{x-2} = +\infty$$

$$\lim_{x\to2^-} \frac{x^2+x}{x^2-2x} = \lim_{x\to2^-} \frac{x+1}{x-2} = -\infty$$

So $\lim_{x\to 2}\dfrac{x^2+x}{x^2-2x}$ DNE.

m) $\displaystyle\lim_{x\to 2^-}\frac{x+3}{x^3-4x^2+4x}=\lim_{x\to 2^-}\frac{x+3}{x(x-2)^2}=+\infty$

n) $\displaystyle\lim_{x\to-\infty}\frac{2x+1}{\sqrt{25x^2+9x}}\overset{\text{HP}}{=}\lim_{x\to-\infty}\frac{2x}{\sqrt{25x^2}}=\lim_{x\to-\infty}\frac{2x}{|5x|}\overset{\text{x<0}}{=}\lim_{x\to-\infty}\frac{2x}{-5x}=-\frac{2}{5}$

o) $\displaystyle\lim_{x\to-\infty}\frac{x^2-9x^3}{x^2+2x-15}\overset{\text{HP}}{=}\lim_{x\to-\infty}\frac{-9x^3}{x^2}=\lim_{x\to-\infty}-\frac{9x}{2}=+\infty$

p) $\displaystyle\lim_{x\to 1}\cos\left(\frac{x^2+x-2}{x^2-x+2}\right)\overset{\text{Cont}}{=}\cos\left(\lim_{x\to 1}\frac{x^2+x-2}{x^2-x+2}\right)\overset{\text{Part(a)}}{=}\cos(0)=1$

q) $\displaystyle\lim_{x\to\pi/2}(e^{\sin x})\overset{\text{Composite: Exp Cont}}{=}e^{\lim_{x\to\pi/2}\sin x}\overset{\text{Trig Cont}}{=}e^{\sin(\pi/2)}=e^1=e$

r) $\displaystyle\lim_{x\to 0^+}\frac{\overbrace{e^x+\cos x}^{1+1}}{\underbrace{\sin^2 x}_{(0^+)^2=0^+}}=+\infty$

s) $\displaystyle\lim_{x\to 1}\frac{2}{x-1}-\frac{2}{x(x-1)}=\lim_{x\to 1}\frac{2x-2}{x(x-1)}=\lim_{x\to 1}\frac{2(x-1)}{x(x-1)}=\lim_{x\to 1}\frac{2}{x}=2.$

4. Which of the following are polynomials? Rational functions? Neither?

a) R **b)** N **c)** P **d)** R **e)** P **f)** R **g)** N **h)** N

5. a) $m_{\text{sec}}=\frac{f(x)-f(3)}{x-3}=\frac{\sqrt{x+1}-2}{x-3}=\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}=\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}=\frac{x-3}{(x-3)(\sqrt{x+1}+2)}=\frac{1}{\sqrt{x+1}+2}$

b) Use the work in part (a): $m_{\tan}=\lim_{x\to 3}m_{\text{sec}}=\lim_{x\to 3}\dfrac{1}{\sqrt{x+1}+2}=\dfrac{1}{4}.$

c) Tangent line slope $m_{\tan}=\frac{1}{4}$. The point is $(3,\sqrt{3+1})=(3,2)$. The line is $y-2=\frac{1}{4}(x-3)$ or $y=\frac{1}{4}x+\frac{5}{4}$.

6. a) $v_{\text{ave}}=\frac{f(x)-f(1)}{x-1}=\frac{(x^2-5x+6)-2}{x-1}=\frac{x^2-5x+4}{x-1}=\frac{(x-1)(x-4)}{x-1}=x-4.$

b) Use the work in part (a): $v_{\text{inst}}\lim_{x\to 1}v_{\text{ave}}=\lim_{x\to 1}x-4=-3.$

c) Secant Slope is to [Tangent Slope] as Average Velocity is to Instantaneous Velocity.

7. a) Use the definition of continuous. Check $f(3)$.... $f(3)=2$ (exists).

b) Check $\displaystyle\lim_{x\to 3}f(x)=\lim_{x\to 3}\frac{x-3}{x^2-2x-3}=\lim_{x\to 3}\frac{x-3}{(x-3)(x+1)}=\lim_{x\to 3}\frac{1}{x+1}=\frac{1}{4}$ exists

c) So f is NOT continuous at $x=3$ because $\lim_{x\to 3}f(x)\neq f(3)$.

8. a) The definition of left continuity says we need $f(1)=z$ to equal $\lim_{x\to 1^-}f(x)$. But

$$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^-}\frac{x^2+2x-3}{x^2+4x-5}=\lim_{x\to 1^-}\frac{(x+3)(x-1)}{(x-1)(x+5)}=\lim_{x\to 1^-}\frac{x+3}{(x+5)}=\frac{2}{3}.$$

We need $z=\frac{2}{3}$.

b) We now need the function to be right-continuous, also. But

$$\lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}\frac{\sqrt{x}-1}{1-x}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1^+}\frac{x-1}{(1-x)(\sqrt{x}+1)}=\lim_{x\to 1^+}\frac{-1}{\sqrt{x}+1}=\frac{-1}{2}.$$

This is not equal to the limit from the left, so the function cannot be continuous at $x=1$.

c) In part (a) we made the function left-continuous at 1; in part (b) we saw it was not right-continuous there. So $(-\infty,1]\cup(1,\infty)$.

9. a) Check $f(0)$.... $f(0)=\frac{3x+4}{x^6+x^2+1}=\frac{4}{2}=4$ (exists). Check $\lim_{x\to 0}f(x)$. Must use 1-sided limits b/c f is "split" at $x=0$. Check

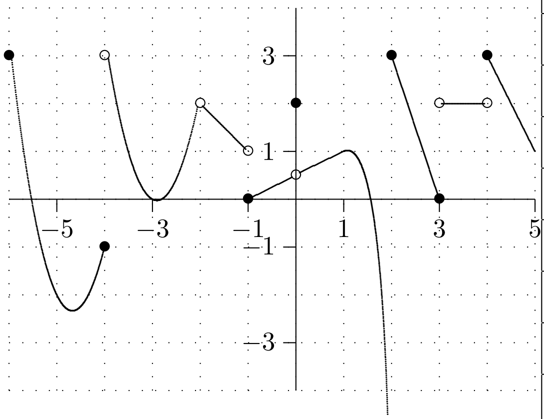
$$\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\frac{3x+4}{x^6+x^2+1}\overset{\text{Rational}}{=}\frac{4}{1}=4$$

$$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}\frac{x}{\sqrt{x+4}-2}\cdot\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}=\lim_{x\to 0^-}\frac{x(\sqrt{x+4}+2)}{x+4-4}=\lim_{x\to 0^-}\frac{x(\sqrt{x+4}+2)}{x}=\lim_{x\to 0^-}\sqrt{x+4}+2\overset{\text{Root}}{=}=4.$$

Since the 1-sided limits are equal, then $\lim_{x\to 0}f(x)=2$. So f is continuous at 2 since $\lim_{x\to 0}f(x)=f(0)$.

b) Near $x=4$ the function is $f(x)=\frac{3x+4}{x^6+x^2+2}$ which is rational with a non-zero denominator, so it is continuous.

10. a)



a	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$	$\lim\limits_{x\to a} f(x)$	$f(a)$	Ctns, Rem
-5	-2	-2	-2	-2	Y, —
-4	-1	3	DNE	-1	L, Not Rem
-2	2	2	2	UND	No, Rem
-1	1	0	DNE	0	R, Not Rem
1	1	1	1	1	Y, —
2	DNE	3	DNE	3	R, Not rem
3	0	2	DNE	0	L, Not Rem
4	Not 3	3	DNE	3	R, Not Rem

b) Find a point a where the limit exists and $f(a)$ exists, but $f(x)$ is not continuous: At $x = 0$.

c) Is f continuous on $[2, 3]$? Yes. On $[-4, -2]$? No. On $(-4, -2)$? Yes.

11. $f(x) = \frac{x^2+7x+10}{x^2+4x-5} = \frac{(x+2)(x+5)}{(x+5)(x-1)}$ is rational and not defined at $x = -5$ and 1. At $x = -5$:

$$\lim_{x\to-5} \frac{x^2+7x+10}{x^2+4x-5} = \lim_{x\to-5} \frac{(x+2)(x+5)}{(x+5)(x-1)} = \lim_{x\to-5} \frac{x+2}{x-1} = \frac{-3}{-6} = \frac{1}{2}.$$

So $x = -5$ is a removable discontinuity, not a VA. At $x = 1$:

$$\lim_{x\to1^+} \frac{x^2+7x+10}{x^2+4x-5} = \lim_{x\to1^+} \frac{(x+2)(x+5)}{(x+5)(x-1)} = \lim_{x\to1^+} \frac{x+2}{x-1} \to \frac{3}{0^+} \to +\infty$$

and

$$\lim_{x\to1^-} \frac{x^2+7x+10}{x^2+4x-5} = \lim_{x\to1^-} \frac{x+2}{x-1} \to \frac{3}{0^-} \to -\infty.$$

In either case, f has a VA at $x = 1$ since a one-sided limit is infinite there. The function is not left or right continuous at either $x = -5$ or 1 because f is not even defined at these points. (Remember the definition.) So the intervals of continuity are $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$.

12. a) To have a removable discontinuity at $x = 0$, the limit must exist there, so both 1-sided limits must be equal.

$$\text{Right}: \lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{x^2+3x}{x} = \lim_{x\to0^+} x + 3 = 3.$$

$$\text{Left}: \lim_{x\to0^-} \frac{x^2-1}{x^2+x} = \lim_{x\to0^-} \frac{(x-1)(x+1)}{x(x+1)} = \lim_{x\to0^-} \frac{x-1}{x} = +\infty.$$

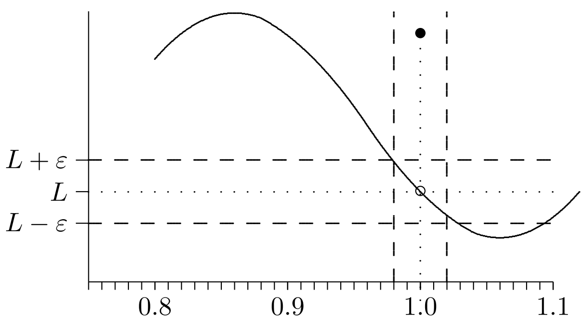
The two 1-sided limits are not equal, so f does not have an RD at $x = 0$.

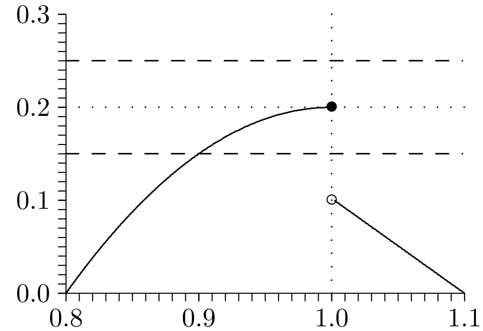
b) Since $\lim_{x\to0^-} = +\infty$ by definition f has a VA at $x = 0$.

13. Remember differentiable implies continuous and NOT continuous implies NOT differentiable.

a	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$	$\lim\limits_{x\to a} f(x)$	$f(a)$	Left Ctns	Right Ctns	Ctns	Removable
-5	2	Not 2	DNE	2	Yes	No	No	No
-4	4	4	4	Not 4	No	No	No	RD
-3	1	2	DNE	2	No	Yes	No	No
-2	-1	-1	-1	4	No	No	No	Yes
-1	4	5	DNE	4	Yes	No	No	No
0	-2	-2	-2	-2	Yes	Yes	Yes	No
1	2	2	2	Not 2	No	No	No	Yes
2	3	3	3	Not 3	No	No	No	Yes
4	$+\infty$	4	DNE	4	No	Right	No	No, VA

14. Note scale! (a) Any δ between 0 and .02 will work. (b) Choose ε with $0 < \varepsilon < 0.1$.





a) Neither is continuous. (a) is not continuous because $\lim_{x\to 1} f(x) \neq f(1)$ and (b) is not continuous because $\lim_{x\to 1} f(x)$ DNE.

b) (a) has a removable discontinuity. Simply make $f(1) = L$ so that $\lim_{x\to 1} f(x) = f(1)$ now.

15. a) Scrap: Find δ. In this case $a = 1$ and $L = 2$. Assume that $\varepsilon > 0$ is given (but arbitrary). Work backwards:

$$|f(x) - L| < \varepsilon \overset{\text{Translate}}{\iff} |(4x+6) - 10| < \varepsilon \overset{\text{Simplify}}{\iff} |4x - 4| < \varepsilon \overset{\text{Factor}}{\iff} 4|x-1| < \varepsilon \overset{\text{Solve}}{\iff} |x-1| < \frac{\varepsilon}{4}.$$

At the last step, we have an inequality of the form $|x - a| < \delta$. We identify δ as $\frac{\varepsilon}{4}$. We are ready to write the actual proof.

Final Proof: Let $\varepsilon > 0$ be given. Assume that $0 < |x - 1| < \frac{\varepsilon}{4}$. Then

$$|(4x+6) - 10| \overset{\text{Simplify}}{=} |4x - 4| \overset{\text{factor}}{=} 4|x-1| < 4 \cdot \frac{\varepsilon}{4} = \varepsilon.$$

b) If $\varepsilon = 0.001$, then $\delta = \varepsilon/4 = 0.00025$.

16. Fill in the exact values (no decimals).

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

17. a) From the LEFT: First $f(0) = 1$.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2+x}{4x} = -\lim_{x\to 0^-} \frac{x+1}{4} \overset{\text{rat'l}}{=} \frac{1}{4}.$$

Since $\lim_{x\to 0^-} f(x) \neq f(0)$, f is NOT left continuous at 0.
From the RIGHT:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\frac{1}{2} - \frac{1}{x+2}}{x} = \lim_{x\to 0^+} \frac{\frac{(x+2)-2}{2(x+2)}}{x} = \lim_{x\to 0^+} \frac{x}{2x(x+2)} = \lim_{x\to 0^+} \frac{1}{2(x+2)} \overset{\text{rat'l}}{=} \frac{1}{4}.$$

But this does NOT equal $f(0)$, so f is NOT right continuous. Since the function is not both left and right continuous, f is not continuous at $x = 0$.

b) Because the left and right limits are equal the two-sided limit exists: $\lim_{x\to 0} f(x) = \frac{1}{4}$. Since $f(0) = 1 \neq \lim_{x\to 0} f(x)$, there is a removable discontinuity at $x = 0$.

c) There is no VA at $x = 0$ because neither 1-sided limit is infinite.

d) Since the function is not left or right continuous at $x = 0$, the intervals are $(-\infty, 0) \cup (0, \infty)$.

18. IVT: The Intermediate Value Theorem. Assume that

- f is continuous on the closed interval $[a, b]$ and
- L is a number between $f(a)$ and $f(b)$.

Then there is at least one number c in (a, b) so that $f(c) = L$.

a) Can the Intermediate Value Theorem be used to guarantee that $f(x) = 7x^4 - 4x^3 - 2x^2 + x = 3$ in the interval $[0, 1]$? We need to check the two hypotheses of the IVT.

- Is $f(x)$ continuous on the interval $[0, 1]$? Yes! Because f is a polynomial.
- Is $L = 3$ between $f(a) = f(0)$ and $f(b) = f(1)$? Well $f(0) = 0$ and $f(1) = 7 - 4 - 2 + 1 = 2$. No, $L = 3$ is NOT between $f(0) = 0$ and $f(1) = 2$.
- So we canNOT apply the IVT to say that there is some number c in $(0, 1)$ so that $f(c) = L = 3$.

b) Can the Intermediate Value Theorem be used to demonstrate that $f(x) = e^x - 3x^2 = 0$ in the interval $[0, 3]$?. **Solution.** Check the hypotheses of the IVT.

- Is $f(x)$ continuous on the interval $[0, 1]$? Yes! Because $f(x) = e^x - 3x^2$ is the difference of continuous functions (a polynomial $3x^2$ and an exponential function e^x), so it is continuous.
- Is $L = 0$ between $f(a) = f(0)$ and $f(b) = f(\pi)$? Well, $f(0) = e^0 - 0 = 1$ and $f(3) = e^3 - 27 \approx -6.91$. So $L = 0$ is between $f(0)$ and $f(3)$.
- So we can apply the IVT to say that there is some number c in $(0, 3)$ so that $f(c) = L = 0$.