Math 130: PracTest 3. Answers Online Friday

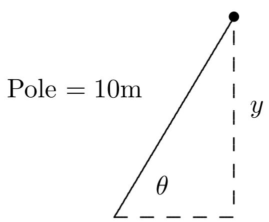
1. Find the absolute extreme values of the following functions on the given intervals. Which theorems justify your work? Make sure you explain what you are doing.

a) $\frac{1}{4}x^4 - x^3 - 2x^2 \quad [1, 5]$ **b)** $\frac{4x}{x^2+1} \quad (-\infty, 0)$ **c)** $3x^{1/3} - x \quad [-8, 8]$

d) $\arctan x - \ln(1 + x^2) \quad (-\infty, \infty)$

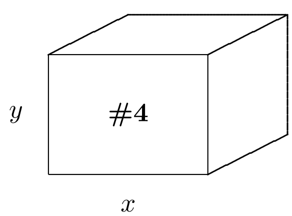
2. a) (From a previous exam) A 10 meter tall flag pole topples to the ground in such a way that the angle θ between the ground and the pole decreases at 0.2 rad/s (see below). How is the distance y between the tip of the flag pole and the ground changing when $y = 8$ meters?

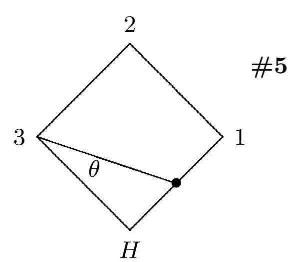
b) How is the 'bottom edge' of the triangle changing at the same moment?



3. (From an exam last year). The photo above shows road salt being unloaded from a tanker at Port Oswego, NY (*NY Times*, 22 Oct 07). The salt comes off the conveyor and forms a conical pile. The height of the pile is changing at 0.1 m/min and the radius at 0.2 m/min. At what rate is the conveyor adding salt to the pile (how is the volume changing) when the pile is 3 m tall and has a radius of 6 m? (Hint: Look up the volume formula for a cone in the front cover of your text, if you need to.)

4. In an animated cartoon, a box with a square base is growing so that its height is increasing at 4 cm/s and its bottom edges are decreasing at 2 cm/s. Determine how the volume of the box is changing when the height is 8 cm and the edge length is 5 cm. Is the volume ($V = x^2h$) increasing or decreasing?





5. a) (See figure above.) From an exam last year: A baseball diamond is a square with 90 ft sides. David Ortiz hits the ball and runs towards first base at a speed of 24 ft/s. At what rate is his distance from **third base** changing when he is halfway to first base?

b) How is the angle θ changing at this same moment?

6. a) Carefully state the Mean Value Theorem and draw a diagram which illustrates it.

b) Carefully state the CIT.

c) State the definitions of a critical point and of an absolute max/min.

7. a) Let $f(x) = x^3 - 3x$ on $[1, 3]$. Does the MVT apply to this function? (Does it satisfy the two assumptions of the MVT? Explain why) **If so, find the point c where** $f'(c) = \frac{f(b)-f(a)}{b-a}$.

b) Does the MVT apply to the function $f(x) = \frac{x+1}{x}$ on $[-2, 3]$? If so, find the point c where $f'(c) = \frac{f(b)-f(a)}{b-a}$. If not, explain why.

8. Draw the graph of a differentiable function on $[0, 5]$ which has no absolute max or explain why this is impossible.

9. Draw a CONTINUOUS function that has a critical number for each set of conditions given, or explain why it is impossible.

a) A critical number at $x = 2$ and $f'(2) \neq 0$.

b) A critical number at $x = 2$ which is not a relative extreme point even though $f'(2) = 0$.

c) A critical number at $x = 2$ where $f'(2)$ DNE which is not a relative extreme point.

10. Do a complete graph of $f(x) = \frac{1}{5}x^5 - \frac{4}{3}x^3$. (To receive full credit on the test, you must indicate critical points, relative extrema, increasing, decreasing, inflections, and concavity. Use number lines for the derivatives.) WARNING: Messy values: Use your calculator to compute function values.

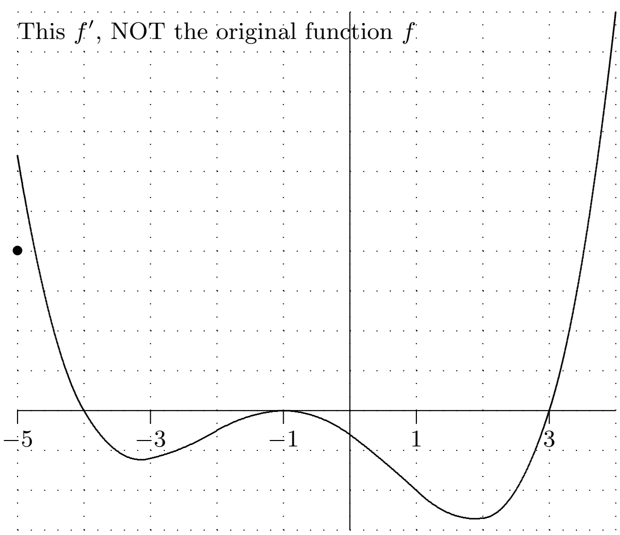
11. Graph $f(x) = xe^x - e^x$. Same directions as above.

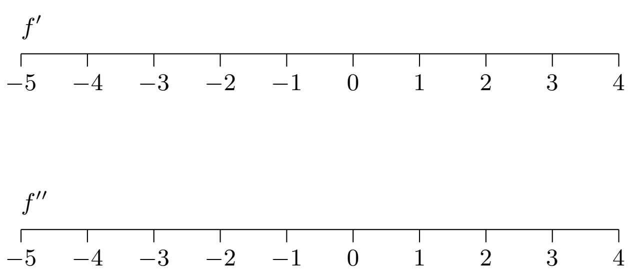
12. The graph below is the graph of $f'(x)$. Draw a possible graph of the original function $y = f(x)$. Determine where there are critical points, relative extremes, points of inflection, where the function is increasing, decreasing, concave up, and concave down. Use number lines to help organize your information.

a) Where is $f'(x) = 0$? Positive? Negative? Mark this on the number line.

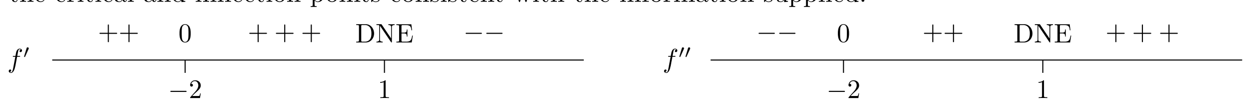
b) Where is $f''(x) = 0$? (You can answer this question by remembering that f'' is the derivative (slope) of f'. So where is the *slope* of the function below 0? Where is it positive? Negative? Mark this on the number line.

c) Now draw the graph of f start at the dot at $(-5, 4)$





13. Use the information about the first and second derivatives of f to sketch a CONTINUOUS function that has derivatives like those given. Indicate on your graph which points are local extrema and which are inflections. Make up y-values for the critical and inflection points consistent with the information supplied.



14. Find the derivatives of the following functions. Use logarithmic differentiation where helpful.

a) $y = (\cos 9x)^x$ **b)** $y = x^{2\tan x}$ **c)** $x^{\ln x}$ **d)** $(\arcsin x)^{x^2}$ **e)** $\ln(4^{7x+1})$

15. Find the derivatives of these functions using the derivative formula for a general exponential function.

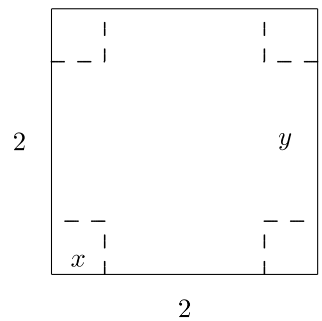
a) $5 \cdot 6^x$ **b)** $x^4 \cdot 4^x$ **c)** $3^{x^2+\tan x}$ **d)** $4^{[x^2+1]^3}$

e) For which values of x does $x^4 \cdot 4^x$ have a horizontal tangent?

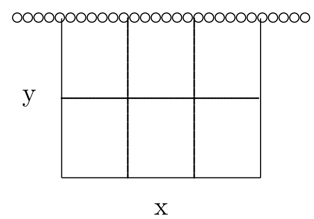
16. Graph $f(x) = 3x^4 - 4x^3$.

17. Another graph: Do a complete graph of $f(x) = (x^2 + 1)e^x$.

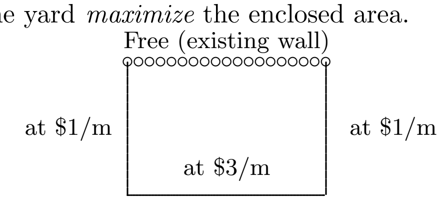
18. A child's sandbox is to be made by cutting equal squares from the corners of a square sheet of galvanized iron and turning up the sides. If each side of the sheet of galvanized iron is 2 meters long, what size squares should be cut from the corners to maximize the volume of the sandbox?



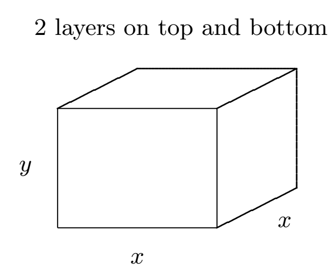
19. A garden is to be created in six sections as shown in the design below using 400 meters of fencing (the solid lines). An existing rock wall is to be used as one side. What is the maximum area that can be enclosed in this design? (Justify your answer.)



20. (From Day 33.) President Gearan has \$600 to spend to enclose his yard to keep out students. One side of the yard will use an existing rock wall while the other 3 sides will use electric fencing. The fence parallel to the wall costs \$3 per meter while the other fence costs \$1 per per meter. What dimensions of the yard *maximize* the enclosed area.



21. (From Day 33.) Most shipping cartons are constructed with double layers of cardboard at the top and bottom (where the carton folds over on itself). If such a carton with a square base is to hold 16,000 cubic cm, what dimensions minimize the material used (surface area, include double top and bottom)?

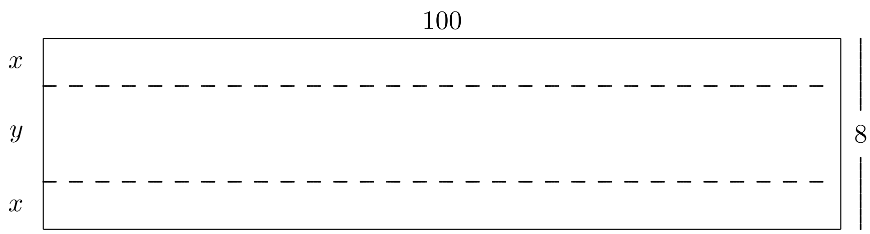


22. A rectangular invitation card is to have an area of 64 square inches. The top and bottom edges are trimmed with gold ink (20 cents per inch) and silver on the sides (5 cents per inch). What dimensions of the rectangle minimize the cost of the ink? Justify your answer.

23. Use the information about the first and second derivatives of f to sketch a function that would have derivatives like those given. Indicate on your graph which points are local extrema and which are inflections. Make up values for the critical and inflection points consistent with the information supplied.

24. A manufacturer makes aluminum cups in the form of right circular cylinders (no top). The volume is to be 8π cu. in. Find the dimensions that use the least material. Hint: Use the volume formula for a cylinder as the constraint equation. The surface area of a cup includes the side and bottom. Justify your answer.

25. A horizontal gutter is to be made from a piece of sheet aluminum 100 inches long and 8 inches wide by folding up equal widths along the dashed lines into a vertical position. How many inches should be turned up on each side to yield the maximum carrying capacity (max volume)? (Ans: 2 inches.)

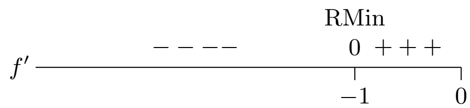


26. Graph $f(x) = \dfrac{6}{x^2+3}$. Indicate everything.

Practest 3A Answers

1. a) Use CIT. $p'(x) = x^3 - 3x^2 - 4x = x(x^2 - 3x - 4) = x(x+1)(x-4)$. Critical #'s at $x = -1, 0, 4$. Only $x = 4$ is in the interval. Check critical #'s: $f(4) = -32$. Endpts: $f(1) = -2.75$ and $f(5) = -18.75$. Abs max at $x = 1$ with $f(1) = -2.75$; Abs min at $x = 4$ with $f(4) = -32$.

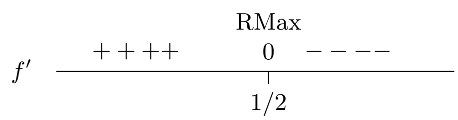
b) Try to use SCPT since the interval is NOT closed. First show that there is only one CP. $f'(x) = \frac{4(x^2+1)-8x^2}{(x^2+1)^2} = \frac{4-4x^2}{(x^2+1)^2} = 0$ at $x = \pm 1$. Only crit # in the interval is $x = -1$. So SCPT applies. Use First Derivative Test (FDT) to classify the point.



There is a relative min at $x = -1$. By the SCPT, there is an absolute min at $x = -1$ with $f(-1) = -2$.

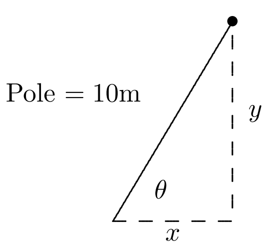
c) Use CIT. $f'(x) = \frac{1}{x^{2/3}} - 1 = 0$ at $x = \pm 1$, DNE at $x = 0$. At crit #'s: $f(-1) = -2$, $f(0) = 0$, $f(1) = 2$. Endpts: $f(-8) = 2$. $f(8) = -2$. Abs max at $x = 2$ and $x = -8$ with $f(2) = f(-8) = 2$; Abs min at $x = -2$ and $x = 8$ with $f(-2) = f(8) = -2$.

d) Try to use SCPT since the interval is NOT closed. First show that there is only one CP. $f'(x) = \frac{1}{1+x^2} - \frac{2x}{1+x^2} = 0$ at $x = \frac{1}{2}$. There is only one CP. So SCPT applies. Use First Derivative Test (FDT) to classify the point.



There is a relative max at $x = 1/2$. By the SCPT, there is an absolute max at $x = 1/2$ with $f(1/2) \approx 0.241$.

2.



Given: $\dfrac{d\theta}{dt} = -0.2$ rad/s.

Find: $\left.\dfrac{dy}{dt}\right|_{y=8}$

Relation: $\frac{y}{10} = \sin\theta$ or $y = 10\sin\theta$.

Rate: $\frac{dy}{dt} = \frac{d}{dt}(10\sin\theta)$.

$$\frac{dy}{dt} = 10\cos\theta\frac{d\theta}{dt}.$$

When $y = 8$, $x = \sqrt{10^2 - 8^2} = 6$ and so $\cos\theta = \frac{6}{10} = 0.6$. Thus,

$$\frac{dy}{dt} = 10(0.6)(-0.2) = -1.2\,\text{m/s}.$$

a) Find $\frac{dx}{dt}$. Relation: $x^2 + y^2 = 10^2 = 100$. So $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ or

$$\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt} = -\frac{8}{6}(-1.2) = 1.6 \text{ m/s}$$

3.

Given: $\frac{dh}{dt} = 0.1$ m/min and $\frac{dr}{dt} = 0.2$ m/min.

Find: $\left.\frac{dV}{dt}\right|_{r=6,\ h=3}$

Relation: $V = \frac{1}{3}\pi r^2 h$.

Rate: $\frac{dV}{dt} = \frac{dy}{dt}(\frac{1}{3}\pi r^2 h)$.

$$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right).$$

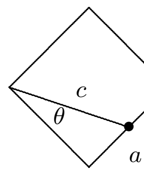
When $r = 6$ and $h = 3$:

$$\left.\frac{dV}{dt}\right|_{r=6,\ h=3} = \frac{\pi}{3}[2(6)(3)(0.2) + 6^2(0.1)] = 3.6\pi \text{ m}^3/\text{min}.$$

4. a) Find $\left.\frac{dV}{dt}\right|_{h=8,\, x=5}$ given $\frac{dh}{dt} = 4$ cm/s and $\frac{dx}{dt} = -2$ cm/s. Relation: $V = x^2 h$. Rate: $\frac{dV}{dt} = 2xh\frac{dx}{dt} + x^2\frac{dh}{dt}$. So

$$\left.\frac{dV}{dt}\right|_{h=8,\, x=5} = 2(5)(8)(-2) + 5^2(4) = -160 + 100 = -60 = \text{cm}^3/\text{s}.$$

5. a) Find $\left.\frac{dc}{dt}\right|_{a=45}$ given $\frac{da}{dt} = 24$ ft/s. Relation: $c^2 = a^2 + 90^2$. Rate: $2c\frac{dc}{dt} = 2a\frac{da}{dt}$ or $\frac{dc}{dt} = \frac{a}{c}\frac{da}{dt}$. When $a = 45$, $c = \sqrt{45^2 + 90^2} = 45\sqrt{5}$. So



$$\frac{dc}{dt} = \frac{45}{45\sqrt{5}}(24) = \frac{24}{\sqrt{5}} \approx 10.733 = \text{ft/s}.$$

b) Now find $\left.\frac{d\theta}{dt}\right|_{a=45}$. Relation (use the constant side!): $\tan\theta = \frac{a}{90}$. Rate: $\sec^2\theta\frac{d\theta}{dt} = \frac{1}{90}\frac{da}{dt}$ or $\frac{d\theta}{dt} = \frac{1}{90\sec^2\theta}\frac{da}{dt} = \frac{\cos^2\theta}{90}\frac{da}{dt}$. Use the triangle: When $a = 45$, from part (a) we have $c = 45\sqrt{5}$, so $\cos^2\theta = (\frac{90}{c})^2 = (\frac{90}{45\sqrt{5}})^2 = (\frac{2}{\sqrt{5}})^2 = \frac{4}{5} = 0.8$. So

$$\left.\frac{d\theta}{dt}\right|_{a=90} = \frac{\cos^2\theta}{90}\frac{da}{dt} = \frac{0.8}{90}(24) = 0.21\overline{3}\,\text{rad/s}.$$

6. a) **MVT: The Mean Value Theorem.** Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there is some point c between a and b so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

b) **CIT:** Let f be a continuous function on a closed interval $[a, b]$. Then the absolute extrema of f occur either at critical points of f on the open interval (a, b) or at the endpoints a and/or b.

c) Assume that f is defined at c. Then c is a **critical number of** f if either (1) $f'(c) = 0$ or (2) $f'(c)$ does not exist.

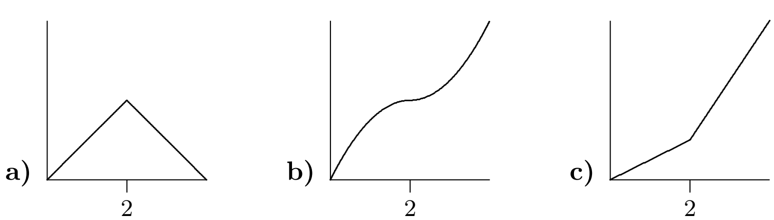
d) Let f be a function whose domain D contains the point c. f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in D. The number $f(c)$ is the **minimum value** of f.

7. a) Yes. Since f is a polynomial it is both continuous and differentiable so the MVT applies to this function. $\frac{f(3)-f(1)}{3-1} = \frac{18+2}{2} = 10$. $f'(x) = 3x^2 - 3 = 10$ or $x^2 = 13/3$. So $x = \pm\sqrt{13/3}$. But only $x = \sqrt{13/3}$ is in the interval.

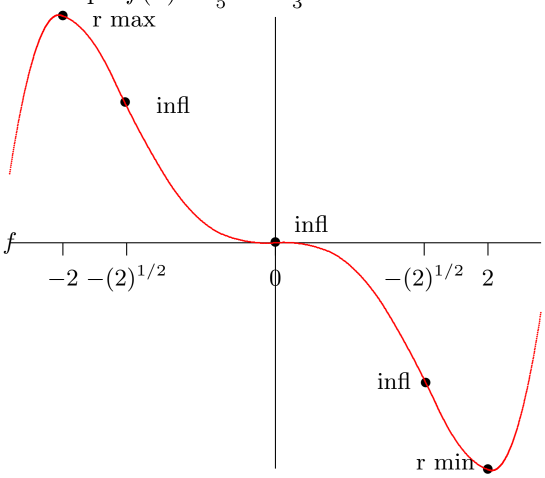
b) No. f is not defined at 0, so f is not continuous.

8. Impossible. Differentiable $\Rightarrow$ continuous. The interval is closed, so EVT $\Rightarrow$ f has an abs max and min.

9. All are possible.



10. Graph $f(x) = \frac{1}{5}x^5 - \frac{4}{3}x^3$.



Locate critical numbers:

$f'(x) = x^4 - 4x^2 = x^2(x^2 - 4)$ at $x = 0, \pm 2$.

	inc		rmax		dec		Neither		dec		rmin		inc
f'	++		0		− − −		0		− − −		0		++
			−2				0				2		

Check concavity with the second derivative.

$f''(x) = 4x^3 - 8x = 4x(x^2 - 2)$ at $x = \pm\sqrt{2}$.

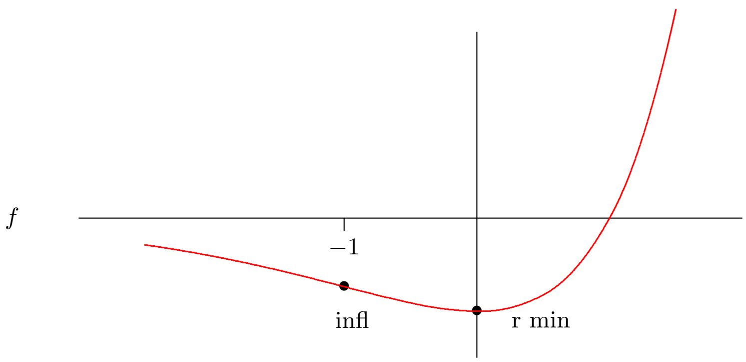
	concave dn	inf	up	inf	dn	inf	concave up
f''	− − −	0	++	0	− −	0	+ + +
		$-(2)^{1/2}$		0		$-(2)^{1/2}$	

Plot the critical numbers and the inflections:

$f(0) = 0$ and $f(2) = -64/15$, $f(-2) = 64/15$.

$f(\sqrt{2}) \approx -2.64$ and $f(-\sqrt{2}) \approx 2.64$.

11. Do a complete graph of $y = f(x) = xe^x - e^x$.



Locate critical numbers:

$f'(x) = xe^x + e^x - e^x = xe^x = 0$ at $x = 0$.

	dec	l min	inc
f'	− − − −	0	+ + ++
		0	

Check concavity with the second derivative.

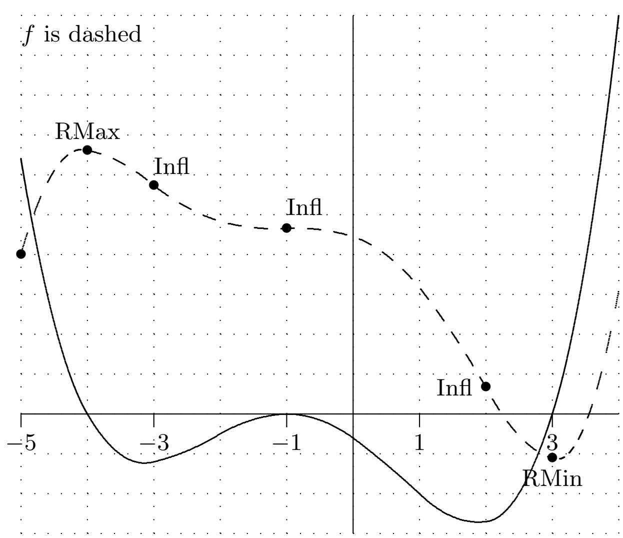
$f''(x) = xe^x + e^x = (x+1)e^x = 0$ at $x = -1$.

	concave dn	inflect	concave up
f''	− − − −	0	+ + ++
		−1	

Plot the critical numbers and the inflections:

$f(0) = -1$ and $f(-1) = -2e^{-1} \approx -0.736$.

12. The graph below is the graph of $f'(x)$. Draw a possible graph of the original function $y = f(x)$.



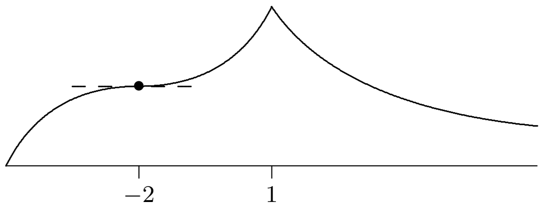
	Inc	RMx	Dec	CP	Dec	RMn	Inc
f'	+	0	− − −	0	− − −	0	++

−5 −4 −3 −2 −1 0 1 2 3 4

	C Dn	Infl	C Up	Infl	C Dn	Infl	C Up
f''	− − −	0	++	0	− − −	0	++

−5 −4 −3 −2 −1 0 1 2 3 4

13. Here's one example:



14. a) $\ln y = \ln(\cos 9x)^x = x\ln(\cos 9x) \implies \frac{1}{y}\cdot\frac{dy}{dx} = \ln(\cos 9x) - \frac{x\sin(9x)\cdot 9}{\cos 9x} \implies \frac{dy}{dx} = (\cos 9x)^x(\ln(\sin 9x) - 9x\tan 9x)$.

b) $\ln y = \ln x^{2\tan x} = 2\tan x \ln x \implies \frac{1}{y}\cdot\frac{dy}{dx} = 2\sec^2 x\ln x + (2\tan x)\frac{1}{x} \implies \frac{dy}{dx} = x^{2\tan x}\left(2\sec^2 x \ln x + \frac{2\tan x}{x}\right)$.

c) $\ln y = \ln x^{\ln x} = \ln x \ln x = (\ln x)^2 \implies \frac{1}{y}\cdot\frac{dy}{dx} = 2(\ln x)\cdot\frac{1}{x} \implies \frac{dy}{dx} = x^{\ln x}\cdot\frac{2\ln x}{x}$.

d) $\ln y = \ln(\arcsin x)^{x^2} = x^2\ln(\arcsin x) \implies \frac{1}{y}\cdot\frac{dy}{dx} = 2x\ln(\arcsin x) + x^2\frac{1}{\sqrt{1-x^2}}$

$$\implies \frac{dy}{dx} = (\arcsin x)^{x^2}\left(2x\ln(\arcsin x) + \frac{x^2}{\sqrt{1-x^2}}\right).$$

e) $y = \ln(4^{7x+1}) = (7x+1)\ln 4 \implies y' = 7\ln 4$.

15. When b is a positive constant, use the derivative rule $\frac{dy}{dx}(b^u) = b^u \ln b \frac{du}{dx}$.

$\frac{d}{dx}[5\cdot 6^x] = 5\cdot 6^x\ln 6 = 5\ln 6(6^x)$.

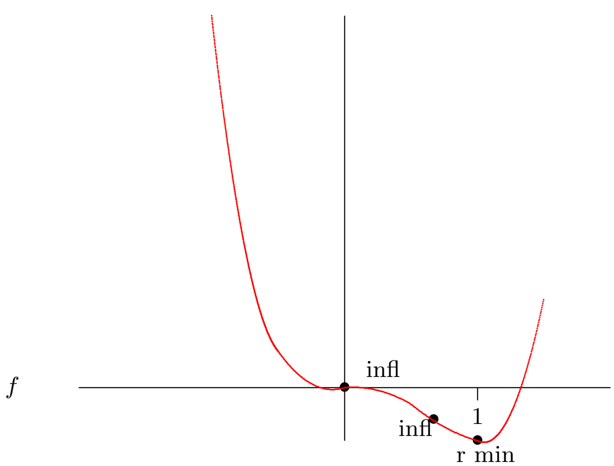
a) $\frac{d}{dx}[x^4\cdot 4^x] = 4x^3\cdot 4^x + x^4\cdot 4^x\ln 4 = x^3\cdot 4^x[4 + x\ln 4]$.

b) $y' = 3^{x^2+\tan x}\cdot\ln 3\cdot(2x + \sec^2 x)$.

c) $y' = 6x[x^2+1]^2 4^{[x^2+1]^3}\ln 4$

d) From part (b), the slope is 0 when $x^3\cdot 4^x[4 + x\ln 4] = 0$. Therefore $x = 0$ or $x = -\frac{4}{\ln 4}$.

16. Graph $f(x) = 3x^4 - 4x^3$.



Locate critical numbers:

$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$ at $x = 0, 1$.

f': dec $- - - -$ | 0 (Neither) | inc $++$ | 0 (rmin) at 1 | inc $++$

Check concavity with the second derivative.

$f''(x) = 36x^2 - 24x = 12x(3x-2) = 0$ at $x = -1$.

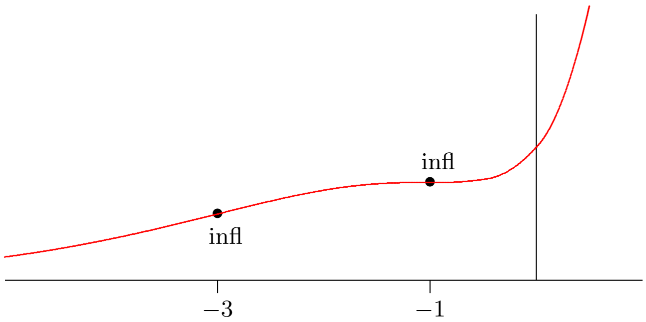
f'': concave up $+ + + +$ | 0 (inf) | dn $- -$ | 0 (inf) at 2/3 | concave up $+ + +$

Plot the critical numbers and the inflections:

$f(0) = 0$ and $f(1) = -1$

$f(2/3) = -16/27 =\approx -0.59851837$.

17. Do a complete graph of $f(x) = (x^2+1)e^x$.



Locate critical numbers: $f'(x) = 2xe^x + (x^2+1)e^x$
$f'(x) = (x^2+2x+1)e^x = (x+1)^2 e^x = 0$ at $x = -1$.

	inc	Neither	inc
f'	$++$	0	$++$
		-1	

Check concavity with the second derivative.
$f''(x) = (2x+2)e^x + (x^2+2x+1)e^x$
$f''(x) = (x^2+4x+3)e^x = (x+1)(x+3)e^x$ at $x = -1,\ -3$.

	conc up	inf	conc dn	inf	conc up
f''	$++$	0	$---$	0	$+++$
		-3		-1	

Plot the critical numbers and inflections:
$f(-1) = 2/e \approx 0.736$ and $f(-3) = 10/e^3 \approx 0.498$.

18. Solution

1. Maximize the area $A = xy$

2. Subject to $400 = 2x + 4y$ where $0 \le x \le 200$.

3. Eliminate y. $y = 100 - \frac{x}{2}$.

4. Rewrite A. $A(x) = x(100 - \frac{x}{2}) = 100x - \frac{x^2}{2}$ on $[0, 200]$.

5. Use CIT (or possibly SCPT). Find the critical numbers. $A'(x) = 100 - x = 0 \implies x = 100$.
Use CIT: Check critical point: $A(100) = 100(50) = 5000\text{m}^2$.
Endpts: $A(0) = 0$ and $A(200) = 0$.
So by CIT, Absolute max at $x = 100$ and area 5000 square meters.

19. Solution

1. Maximize the volume $V = xy^2$

2. Subject to $2 = 2x + y$ where $0 \le x \le 1$.

3. Eliminate y. $y = 2 - 2x$.

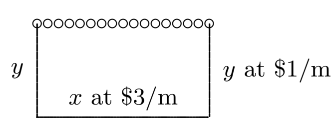
4. Rewrite V. $V(x) = x(2-2x)^2 = 4x - 8x^2 + 4x^3$ on $[0, 1]$.

5. Use CIT (or possibly SCPT). Find the critical numbers.

$$V'(x) = 4 - 16x + 12x^2 = 4(1 - 4x + 3x^2) = 4(1-x)(1-3x) = 0 \implies x = 1,\ 1/3.$$

Use the CIT.
$V(1/3) = \frac{1}{3}(\frac{4}{3})^2 = \frac{16}{27}$.
$V(0) = 0 = V(1)$.
So by the CIT, the absolute max occurs when $x = 1/3$ and the volume is $\frac{16}{27}$ cubic meters.

20. Maximize: Area $A = xy$



Constraint: Cost $C = 3x + 2y = 600$

Eliminate: $y = 300 - \frac{3}{2}x$ Smallest $x = 0$, largest $x = 200$ since $3(200) = 600$

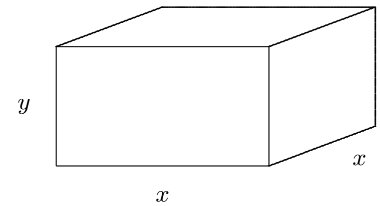
Substitute: $A = x(300 - \frac{3}{2}x) = 300x - \frac{3}{2}x^2$ Domain: $[0, 200]$

Differentiate: $A' = 300 - 3x = 0 \Rightarrow x = 100$

Justify: Since the interval is closed, we can use the CIT. Evaluate A at the critical and end points.
At the Critical point: $A(100) = 100(300 - \frac{3}{2}100) = 15,000$ (Abs Max).
At the End points: $A(0) = 0$, $A(200) = 0$.
So by the CIT, the Abs Max of $15,000$ square meters occurs at $x = 100$ m and $y = 300 - \frac{3}{2}100 = 150$ m.

21.

2 layers on top and bottom



Minimize: Materials $S = \overbrace{4x^2 + 4xy}^{\text{2 tops, 2 bottoms + 4 sides}}$

Constraint: Volume $V = x^2y = 16000$

Eliminate: $y = \frac{16000}{x^2}$ — y can be any value larger than 0

Substitute: $S = 4x^2 + \frac{64000}{x}$ — Domain: $(0, \infty)$

Differentiate: $S' = 8x - \frac{64000}{x^2} = 0 \Rightarrow 8x^3 = 64000$ — So $x^3 = 8000$ so $x = 20$

Justify: There is only one CP, so use the SCPT. Classify the CP with the First Derivative Test.

S': on $(0, 20)$: $- - -$; at 20: R Min; on $(20, \infty)$: $+ + +$

There is a Relative Min at $x = 20$ which must be an Abs Min by SCPT. $y = 40$.

22.

Minimize: Cost $C = 20(2x) + 5(2y)$ — $20\times$ (top and bottom) plus 5×2 sides

Constraint: Area $A = xy = 64$

Eliminate: $y = \frac{64}{x}$ — Domain: $0 < x < \infty$

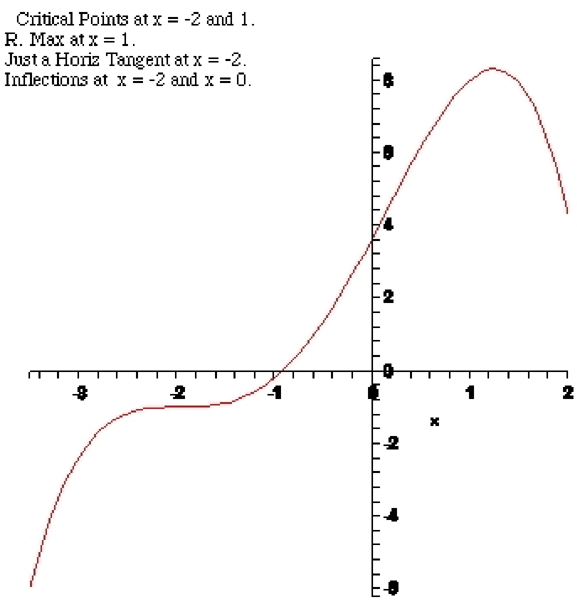
Substitute: $C = 40x + \frac{640}{x}$ — $(0, \infty)$

Differentiate: $C' = 40 - \frac{640}{x^2} = 0 \Rightarrow 4x^2 = 640$ — $\Rightarrow x^2 = 16$ so $x = 4$, $x \neq -4$

Justify: 1st Der Test C': on $(0, 4)$: $- - -$; at 4: 0; after 4: $+ + +$ — R. Min at $x = 4$ is an Abs Min by SCPT

since there is only one critical point and it is a rel min. $y = 16$.

23.



24.

Minimize: $SA = \pi r^2 + 2\pi r h$cr Constraint: — Vol $V = \pi r^2 h = 8\pi$

Eliminate: $h = \frac{8\pi}{\pi r^2} = \frac{8}{r^2}$ — Domain: $0 < r < \infty$

Substitute: $SA = \pi r^2 + \frac{16\pi}{r}$ — $(0, \infty)$

Differentiate: $SA' = 2\pi r - \frac{16\pi}{r^2} = 0 \Rightarrow r^3 = 8$ — $\Rightarrow r = 2$

Justify: 1st Der Test SA': on $(0, 3)$: $- - -$; at 3: 0; after: $+ + +$ — R. Min at $r = 2$ in. is an Abs Min by SCPT

since there is only one critical point and it is a rel min. $h = 2$ in.

25. Maximize: $V = 100xy$

Constraint: $2x + y = 8$

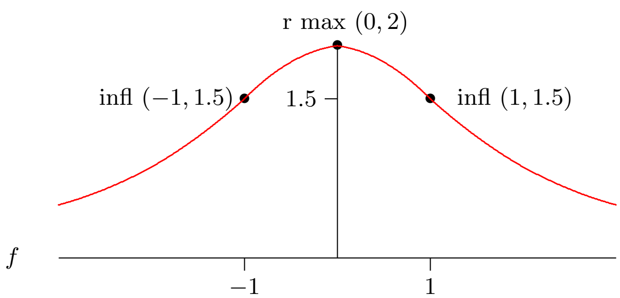
Eliminate: $y = 8 - 2x$ Smallest $x = 0$, largest $x = 4$ since $2(4) = 8$

Substitute: $V = 100x(8 - 2x) = 800x - 200x^2$ Domain: $[0, 4]$

Differentiate: $V' = 800 - 400x = 0 \Rightarrow x = 2$

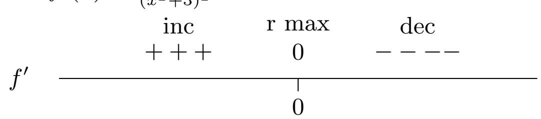
Justify: Since the interval is closed, we can use the CIT. Evaluate V at the critical and end points.
At the Critical point: $V(2) = 1600 - 800 = 800$.
At the End points: $V(0) = 0$, $V(4) = 0$.
So by the CIT, the Abs Max of 800 square inches occurs at $x = 2$ in and $y = 4$ in.

26. Graph $f(x) = \dfrac{6}{x^2 + 3}$.



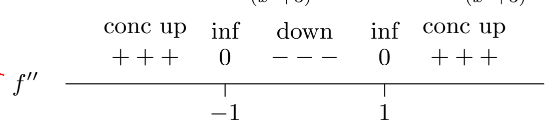
Locate critical numbers:

$f'(x) = \frac{-12x}{(x^2+3)^2}$ at $x = 0$.



Check concavity with the second derivative.

$f''(x) = \frac{-12(x^2+3)^2 + 12x(2)(x^2+3)(2x)}{(x^2+3)^4} = \frac{36x^2 - 36}{(x^2+3)^3} = 0$ at $x = \pm 1$.



Plot the critical numbers and the inflections:

$f(0) = 2$ and $f(-1) = f(1) = 1.5$