

## Calc 1 Review

Things to Memorize, Derivative Laws

- You should have all of the following derivative laws memorized

$$\Rightarrow \frac{d}{dx}(x^n) = nx^{n-1}, \quad \frac{d}{dx}(u^n) = n u^{n-1} \frac{du}{dx} \quad \text{where } u \text{ is a function of } x$$

$$\Rightarrow \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x), \quad \frac{d}{dx}(cF(x)) = c \frac{d}{dx}(F(x)) = c f'(x)$$

$$\Rightarrow \frac{d}{dx}(f(g(x))) = f'(g(x))g'(x), \quad \frac{dF}{dx} = \frac{dF}{du} \frac{du}{dx} \quad (\text{know/understand 1 of these})$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\Rightarrow \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)) = f'(x) + g'(x)$$

Things to Memorize, Simple Derivatives

- You should have all of these derivatives memorized, and know how to use them in the context of the above laws, especially chain rule

$$\Rightarrow \frac{d}{dx}(x^n) = nx^{n-1}, \quad \frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\Rightarrow \frac{d}{dx}(b^x) = \ln(b) b^x, \quad \frac{d}{dx}(\sin(x)) = \cos(x), \quad \frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\Rightarrow \frac{d}{dx}(\tan(x)) = \sec^2(x), \quad \frac{d}{dx}(\sec(x)) = \sec(x) \tan(x)$$

$$\Rightarrow \frac{d}{dx}(\cot(x)) = -\operatorname{csc}^2(x), \quad \frac{d}{dx}(\csc(x)) = -\csc(x) \cot(x)$$

$$\Rightarrow \frac{d}{dx}(\underbrace{\sin^{-1}(x)}_{\text{some function}}) = \frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{d}{dx}(\cos^{-1}(x)) = \frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{d}{dx}(\tan^{-1}(x)) = \frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

} you should have some idea of how you find these derivatives using implicit differentiation

## Calc 1 Review

All Things trig

- if possible, know all of this forward & backward

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	DNE

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

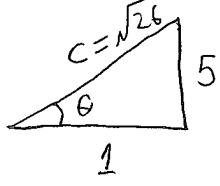
$$\sec \theta = \frac{1}{\cos \theta}, \csc \theta = \frac{1}{\sin \theta}$$

$\Rightarrow$  Trig Derivatives are on page 1. Memorize Them

Example:  $\sin(\arctan(5)) \Rightarrow$  always start by letting

$\theta = \arctan(5) \Rightarrow$  take  $\tan()$  of both sides

$$\tan \theta = 5 = \frac{5}{1} \Rightarrow \text{draw triangle so } \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{1}$$



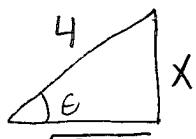
$$C^2 = 5^2 + 1^2 \Rightarrow C^2 = 26 \Rightarrow C = \sqrt{26}$$

so  $\sin(\overbrace{\arctan(5)}) = \sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{5}{\sqrt{26}}$  so

$$\boxed{\sin(\arctan(5)) = 5/\sqrt{26}}$$

Example  $\sin^{-1}(\sqrt{3}/2) \Rightarrow$  let  $\theta = \sin^{-1}(\sqrt{3}/2)$ , so  $\sin(\theta) = \frac{\sqrt{3}}{2}$   
 so using the chart above, find  $\theta$  so that  $\sin \theta = \frac{\sqrt{3}}{2}$   
 $\theta = \pi/3 = \sin^{-1}(\sqrt{3}/2)$

Example  $\cos(\arcsin(x/4)) \Rightarrow \theta = \arcsin(x/4) \Rightarrow \sin(\theta) = \frac{x}{4}$



$$\text{so } \cos(\theta) = \frac{\sqrt{4^2 - x^2}}{4} \Rightarrow \boxed{\frac{\sqrt{4^2 - x^2}}{4} = \cos(\arcsin(x/4))}$$

$\sqrt{4^2 - x^2} \rightarrow$  from pythagorean thm.

## Calc 1 Review

Limit definition of the derivative

⇒ The formal definition of the derivative is the limit definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \text{know this, you will have to use it}$$

Tangent lines

⇒ know how to write the equation of the tangent to a curve at a point

⇒  $m_{\text{tan}} = f'(a)$ , tangent line uses point slope form

$$y - y_1 = m_{\text{tan}}(x - x_1) \text{ where } (x_1, y_1) \text{ is the point you want the}$$

line to go through ⇒  $x_1 = a$ ,  $y_1 = f(a)$ , so in general we have

$$y - f(a) = f'(a)(x - a)$$

Example: Tangent line to the curve  $y = 2x^2 + 5x$  at  $\overset{a=3}{x=3}$

$$\Rightarrow f'(x) = 4x + 5 \text{ so } m_{\text{tan}} = f'(3) = 4(3) + 5 = 17, \text{ line must}$$

go through  $(3, f(3))$ ,  $f(3) = 2(3^2) + 5(3) = 33$ , so we have

$$\boxed{y - 33 = 17(x - 3)} \Rightarrow \text{leave in this form, don't unnecessarily simplify and make an algebra mistake}$$

Alternative Definitions

know these definitions / how they fit together with the above

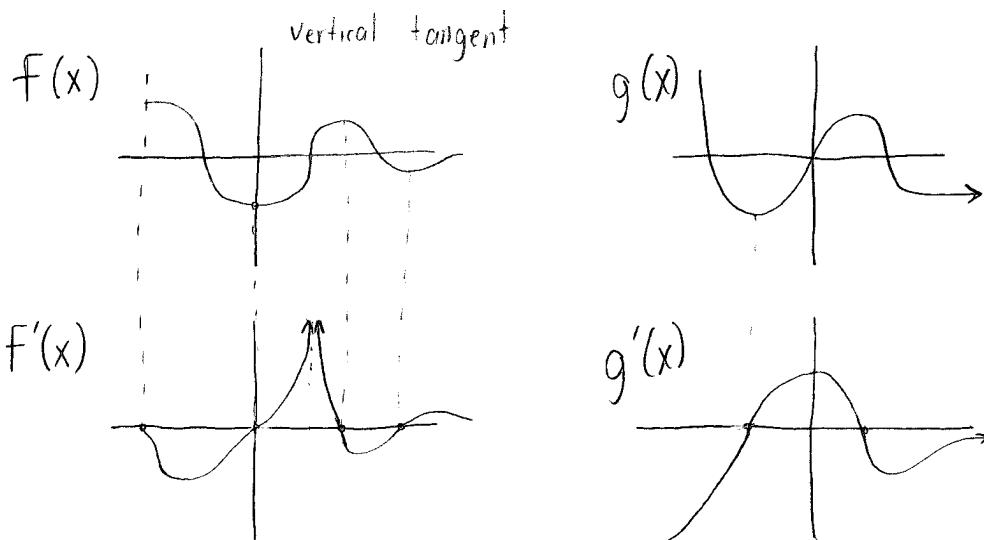
$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}, m_{\text{tan}} = \lim_{x \rightarrow a} (m_{\text{sec}}) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

## Calc 1 Review

Sketching Derivatives

Given a graph of  $f(x)$ , be able to sketch a graph of  $f'(x)$

Example

Note

⇒ when  $f(x)$  is flat, then  $f'(x) = 0$

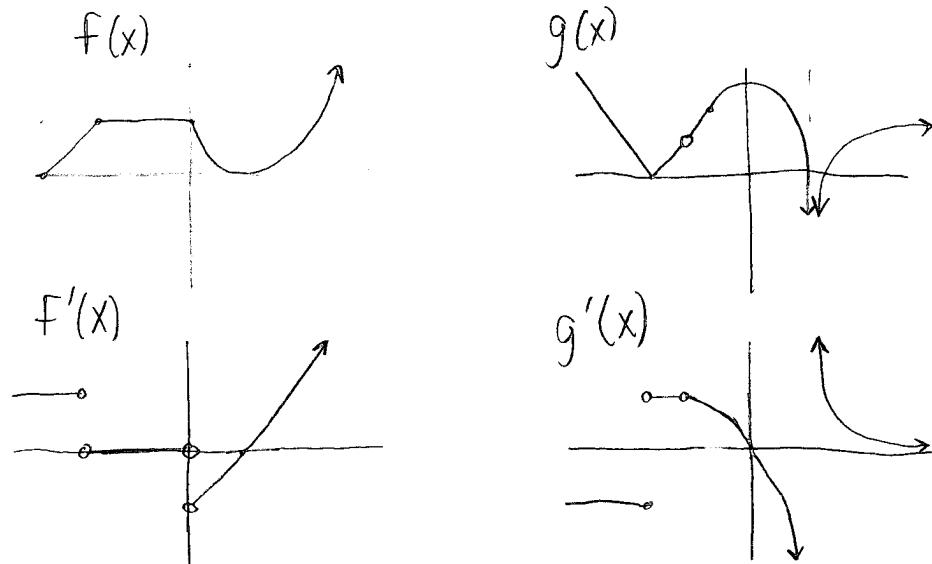
⇒  $f'(x)$  is positive when  $f(x)$  has a positive slope / is increasing

⇒  $f'(x)$  is negative when  $f(x)$  has a negative slope / is decreasing

⇒ if  $f(x)$  has a vertical tangent,  $f'(x)$  will have a vertical asymptote

More Examples

\* these graphs don't have to be exact, but must capture all the main features of  $f'(x)$



\* differentiability implies continuity but continuity does not imply differentiability \*

## Calc 1 Review

Combinations of Derivative rules

→ know how to combine derivative rules when needed, best way to avoid making a mistake is to write everything out

Ex:  $\frac{d}{dx}(\sin^3(x^2 \cos(x))) \Rightarrow$  chainrule and product rule

$$f(x) = \sin^3(x^2 \cos(x)) \Rightarrow \text{let } u = \sin(x^2 \cos(x)), \text{ then}$$

$$f(u) = u^3 \Rightarrow \frac{df}{du} = 3u^2, \text{ now find } \frac{du}{dx} \Rightarrow \frac{d}{dx}(\sin(x^2 \cos(x)))$$

$$\text{let } v = x^2 \cos(x) \Rightarrow u(v) = \sin(v) \Rightarrow \frac{du}{dv} = \cos(v), \text{ now find } \frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{d}{dx}(x^2 \cos(x)) \Rightarrow \text{product rule} \Rightarrow \frac{dv}{dx} = 2x \cos(x) + (-\sin(x))x^2$$

$$\text{now putting it all together } \frac{df}{dx} = \frac{df}{du} \frac{du}{dv} \frac{dv}{dx} = (3u^2)(\cos(v))(2x \cos(x) - x^2 \sin(x))$$

plugging in for  $u, v \Rightarrow \boxed{\frac{df}{dx} = 3 \sin^2(x^2 \cos(x)) \cos(x^2 \cos(x))(2x \cos(x) - x^2 \sin(x))}$

Alternative Method  $\frac{d}{dx}(\sin^3(x^2 \cos(x))) \Rightarrow$  write  $\sin^3(x^2 \cos(x))$  as  $f(g(h(x)))$

$$f(x) = x^3, g(x) = \sin(x), h(x) = x^2 \cos(x) \Rightarrow \text{find } f', g', \text{ & } h'$$

$$f'(x) = 3x^2, g'(x) = \cos(x), h'(x) = \frac{d}{dx}(x^2 \cos(x)) = 2x \cos(x) + x^2(-\sin(x)) \Rightarrow \text{plug into}$$

$$\text{chainrule } \frac{d}{dx}(f(g(h(x)))) = \underbrace{f'(g(h(x)))}_{\frac{df}{du}} \underbrace{g'(h(x))}_{\frac{du}{dv}} \underbrace{h'(x)}_{\frac{dv}{dx}}$$

$$\frac{d}{dx}(\sin^3(x^2 \cos(x))) = \underbrace{3(\sin(x^2 \cos(x)))^2}_{\frac{df}{du}} \underbrace{(\cos(x^2 \cos(x)))}_{\frac{du}{dv}} \underbrace{(2x \cos(x) - x^2 \sin(x))}_{\frac{dv}{dx}}$$

- \* you must know 1 of these approaches. They are equivalent
- \* you don't have to show as much work as I did, but it will help you avoid mistakes and get partial credit\*

\* you will not be asked anything this hard. this problem demonstrates the process you would have to do if you were asked something like this \*

## Calc 1 Review

### Combinations of Derivative rules cont.

Example:  $\frac{d}{dx} \left( \frac{\cos(x^2) \arcsin(3x^5)}{\tan(\ln(x))} \right) \Rightarrow$  this is much harder than anything on your exam, but you can do it

let  $f(x) = \cos(x^2) \arcsin(3x^5)$ ,  $g(x) = \tan(\ln(x))$ , find  $F'$  &  $g'$

$\frac{d}{dx} (\cos(x^2) \arcsin(3x^5)) \Rightarrow$  product & chain rule at the same time

$$\frac{dF}{dx} = (-\sin(x^2))(2x) \arcsin(3x^5) + \cos(x^2) \left( \frac{1}{\sqrt{1-(3x^5)^2}} \right) (15x^4)$$

$$\frac{dg}{dx} (\tan(\ln(x))) \Rightarrow \text{chain rule} \Rightarrow \frac{dg}{dx} = \sec^2(\ln(x)) \left( \frac{1}{x} \right)$$

now plug into quotient rule  $\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

ran out of room

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{\left( -2x \sin(x^2) \arcsin(3x^5) + \cos(x^2) \left( \frac{1}{\sqrt{1-(3x^5)^2}} \right) (15x^4) \right) \tan(\ln(x)) - \overbrace{\left( f(x) g'(x) \right)}_{\text{would plug in}}}{(\tan(\ln(x)))^2}$$

where  $F(x)g'(x) = \cos(x^2) \arcsin(3x^5) \sec^2(\ln(x)) \left( \frac{1}{x} \right)$

$\Rightarrow$  this problem may have an incredibly long answer, but if you are careful, write things out explicitly, you could take this derivative

$\Rightarrow$  practice these!! you learn by doing, so do as many of these types of problems as you can

## Calc 1 Review

Implicit Differentiation (Theory)

⇒ know how this works, and be able to explain in your own words why it works

My Explanation

imagine  $v(t) = 3t^4 + e^{4t}$ , and we want  $\frac{d}{dt}(v^3)$   
 $\frac{d}{dt}(v^3) = \frac{d}{dt}((3t^4 + e^{4t})^3) = (\overbrace{3(3t^4 + e^{4t})^2}^{\text{3 pieces}})(\overbrace{12t^3 + 4e^{4t}}^{\frac{dv}{dt}})$

⇒ since we explicitly knew  $v(t)$ , we could just plug in and take a derivative. this is "explicit" differentiation

now imagine we don't know exactly what  $v(t)$  is, just that it is some function of  $t$ . To be consistent with above, the same pieces of  $\frac{d}{dt}(v^3)$  must be present

$\frac{d}{dt}(v^3) = 3v^2 \frac{dv}{dt} \Rightarrow$  If we had  $v(t)$  explicitly, we could plug in for each piece above. But since we don't have  $v(t)$  exactly, this is the best we can do

Implicit differentiation works the same way. We don't have  $y(x)$  exactly, we just know  $y$  is some function of  $x$

so  $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx} \Rightarrow$  this is the best we can do

⇒ Once we've differentiated both sides, we use algebra to solve for  $\frac{dy}{dx}$  carefully watch out for order of operations

Alternative Method: Since  $\frac{dF}{dx} = \frac{dF}{du} \frac{du}{dx}$ , we can say  $\frac{d}{dx}(\ ) = \frac{dy}{dx} \frac{d}{dy}(\ )$

so  $\frac{d}{dx}(y^3) = \frac{dy}{dx} \frac{d}{dy}(y^3) = \frac{dy}{dx}(3y^2) = 3y^2 \frac{dy}{dx}$ , same as before

## Calc 1 Review

Implicit Differentiation (Examples)

1.  $y^2 + 2x \sin(y) = x^3$ , Find  $\frac{dy}{dx}$   $\Rightarrow$  take  $\frac{d}{dx}$  on both sides

$$\Rightarrow 2y \frac{dy}{dx} + 2\sin(y) + 2x\cos(y) \frac{dy}{dx} = 3x^2 \Rightarrow \text{now isolate } \frac{dy}{dx}$$

$$2y \frac{dy}{dx} + 2x\cos(y) \frac{dy}{dx} = 3x^2 - 2\sin(y) \Rightarrow \text{Factor out } \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y + 2x\cos(y)) = 3x^2 - 2\sin(y) \Rightarrow \boxed{\frac{dy}{dx} = \frac{3x^2 - 2\sin(y)}{2y + 2x\cos(y)}}$$

2.  $e^{xy^2} = \ln(y)$   $\Rightarrow e^{xy^2}(y^2 + x(2y \frac{dy}{dx})) = \frac{1}{y} \frac{dy}{dx}$

$$e^{xy^2} y^2 + e^{xy^2} 2xy \frac{dy}{dx} = \frac{1}{y} \frac{dy}{dx} \Rightarrow \frac{1}{y} \frac{dy}{dx} - e^{xy^2} 2xy \frac{dy}{dx} = e^{xy^2} y^2$$

$$\Rightarrow \frac{dy}{dx} (\frac{1}{y} - e^{xy^2} 2xy) = e^{xy^2} y^2 \Rightarrow \boxed{\frac{dy}{dx} = \frac{e^{xy^2} y^2}{(\frac{1}{y} - e^{xy^2} 2xy)}}$$

\* do as little algebra as necessary. Once you have  $\frac{dy}{dx}$   
don't simplify unless the question specifically asks you to  
the more you simplify, the more chances you have to make  
a mistake \*

3.  $\arcsin(y) = \arctan(y^2) + x$   $\Rightarrow \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{1+(y^2)^2} (2y \frac{dy}{dx}) + 1$

$$\text{so } \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} - \frac{1}{1+y^4} (2y) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} \left( \frac{1}{\sqrt{1-y^2}} - \frac{2y}{1+y^4} \right) = 1 \Rightarrow \boxed{\frac{dy}{dx} = 1 / \left( \frac{1}{\sqrt{1-y^2}} - \frac{2y}{1+y^4} \right)}$$

\* don't simplify this!! \*

## Calc 1 Review

Logarithmic Differentiation

- this is when you take the log of both sides and then use implicit differentiation
- only need to use when we have  $x$  in both the base and the exponent.  $y = f(x)^{g(x)}$

Examples

$$y = 7^x \Rightarrow \text{dont use!} \text{ use instead } \frac{dy}{dx}(b^x) = \ln(b) b^x \quad \leftarrow$$

$$y = 5^{\cos(x)} \Rightarrow \text{dont use!} \text{ use instead } \frac{dy}{dx}(b^u) = \ln(b) b^u \frac{du}{dx} \quad * \begin{matrix} \text{chain rule} \\ \text{version of} \end{matrix}$$

$$y = (\sin(x))^{\cos(x)} \Rightarrow \text{now use Logarithmic Differentiation}$$

$$\ln(y) = \ln((\sin(x))^{\cos(x)}) = \underbrace{\cos(x) \ln(\sin(x))}_{\text{product rule}} \Rightarrow \text{now take } \frac{d}{dx}$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(\cos(x) \ln(\sin(x)))$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (-\sin(x)) \ln(\sin(x)) + \cos(x) \frac{1}{\sin(x)} \cos(x) \Rightarrow \text{multiply by } y$$

$$\boxed{\frac{dy}{dx} = \left( \cos^2(x) \frac{1}{\sin(x)} - \sin(x) \ln(\sin(x)) \right) (\sin(x))^{\cos(x)}}$$

\* use log rules to simplify expression before taking  $\frac{d}{dx}$

Log Rules

$$\boxed{\ln(AB) = \ln(A) + \ln(B)},$$

$$\boxed{\ln(A/B) = \ln(A) - \ln(B)}$$

$$\boxed{\ln(A^n) = n \ln(A), \text{ but } (\ln(A))^n \neq n \ln(A)}$$

$$\text{Ex: } \frac{d}{dx} \left( \ln \left( \frac{x^2+x}{\sin(x)} \right) \right) = \frac{d}{dx} \left( \ln(x^2+x) - \ln(\sin(x)) \right)$$

$$= \left[ \frac{1}{x^2+x} (2x+1) - \frac{1}{\sin(x)} (\cos(x)) \right] \Rightarrow \text{log rules let you avoid a nasty chain \& Quotient rule}$$

## Calc 1 Review

General Strategies / avoiding common mistakes

1. Before taking  $\frac{dy}{dx}$ , try and simplify the expression by using log rules, trig identities, etc. this will make  $\frac{dy}{dx}$  much easier to take
  2. With Implicit differentiation, you only get a  $\frac{dy}{dx}$  when the derivative "hits" a  $y$ -function. This is common with the chain rule. Ex  $\frac{d}{dx}(x^2 y^3) = \underbrace{\frac{d}{dx}(x^2)}_{\text{no } \frac{dy}{dx}} y^3 + x^2 \underbrace{\frac{d}{dx}(y^3)}_{\frac{dy}{dx} \text{ comes out}} = 2x y^3 + x^2 y^2 \frac{dy}{dx}$
  3. Quotient rule sucks. try and avoid it by simplifying whenever possible
  4. Before taking a complicated derivative, plan out which rules you must use ahead of time. To avoid mistakes, write out everything. This will help you get partial credit
  5. product and chain rule together are very common  
avoid the following mistake this is what not to do
- don't do this  $\frac{d}{dx}((e^{7x} x^3)^3) \neq 3(7e^{7x} 3x^2)$ , i.e.  $\frac{d}{dx}((F(x) \cdot g(x))^3) \neq 3(F'(x)g'(x))$
- do this instead, do  $\frac{d}{dx}((e^{7x} x^3)^3) = 3(e^{7x} x^3)^2 (7e^{7x} x^3 + e^{7x} (3x^2))$
- do this and  $\frac{d}{dx}((F(x) \cdot g(x))^3) = 3(F(x) \cdot g(x))^2 (F'(x)g(x) + g'(x)F(x))$
6. Once you have  $\frac{dy}{dx}$ , don't unnecessarily simplify unless otherwise told to
  7. know your trig and log rules as well as possible