5-Minute Review: Conjugates

Conjugates are usually discussed in reference to expressions involving square roots and typically have the form $a+\sqrt{b}$ and $a-\sqrt{b}$, where a can be any expression. For example, $\sqrt{x}+\sqrt{3}$ and $\sqrt{x}-\sqrt{3}$. Conjugates are useful because when you multiply them together, the 'middle terms' cancel: e.g.,

$$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$$

or

$$(\sqrt{x}+\sqrt{3})(\sqrt{x}-\sqrt{3}) = x-3.$$

EXAMPLE 4.0.1. Use conjugates to simplify the following expression: $\dfrac{4}{\sqrt{x+2}-\sqrt{x}}$.

Solution. Multiply both the numerator and denominator by the conjugate:

$$\frac{4}{\sqrt{x+2}-\sqrt{x}}\cdot\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}} = \frac{4(\sqrt{x+2}+\sqrt{x})}{x+2-x} = \frac{4(\sqrt{x+2}+\sqrt{x})}{2} = 2(\sqrt{x+2}+\sqrt{x})$$

EXAMPLE 4.0.2. Use conjugates to simplify the following expression: $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$.

Solution. Multiply both the numerator and denominator by the conjugate:

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

YOU TRY IT 4.1. Simplify each of these expressions by using an appropriate conjugate.

(a) $\dfrac{x-5}{\sqrt{x}-\sqrt{5}}$ (b) $\dfrac{2x-18}{\sqrt{x}-3}$ (c) $\dfrac{\sqrt{8+h}-\sqrt{8}}{h}$

ANSWER TO YOU TRY IT 4.1. (a) $\sqrt{x}+5$; (b) $2(\sqrt{x}+3)$; (c) $\frac{1}{\sqrt{8+h}+\sqrt{8}}$

☞ See how conjugates are used to calculate limits in the Day 5 (scroll to page 12): `http://math.hws.edu/~mitchell/Math130F16/tufte-latex/Day05.pdff`.