Prerequisites for Math 130

The material below represents only **some** of the basic material with which you should be familiar. We will **not** be reviewing this material. You may wish to consult Appendix A in your your text as well as parts of Chapter 1.

1. Exponents and radicals, including:

a) $x^m x^n = x^{m+n}$ **b)** $(x^m)^n = x^{mn}$ **c)** $\sqrt[n]{x} = x^{1/n}$

d) $\dfrac{1}{x^n} = x^{-n}$ **e)** $\sqrt[n]{x^m} = x^{m/n}$

2. The quadratic formula: If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

3. Interval notation (note the different types of brackets):

a) Open interval: $a < x < b$ or (a, b).

b) Closed interval: $a \leq x \leq b$ or $[a, b]$.

c) Half-open interval: $a \leq x < b$ or $[a, b)$ and $a < x \leq b$ or $(a, b]$.

d) Various rays: $x < a$ or $(-\infty, a)$; $a \geq x$ or $[a, \infty)$.

e) All reals: $(-\infty, \infty)$.

4. Absolute value: If $a > 0$, then:

a) $|x| = a$ means $x = \pm a$.

b) $|x| < a$ means $-a < x < a$ or $(-a, a)$.

c) $|x| \leq a$ means $-a \leq x \leq a$ or $[-a, a]$.

d) $|x| > a$ means $x < -a$ or $x > a$ or $(-\infty, -a) \cup (a, \infty)$.

e) $|x| \geq a$ means $x \leq -a$ or $x \geq a$ or $(-\infty, -a] \cup [a, \infty)$.

f) Note that $\sqrt{x^2} = |x|$ not just x. (Try a negative value for x to see why.)

5. a) The expression $|x - a|$ represents the *distance* between x and a. So for example, $|x - 2| = 3$ says that the distance between x and 2 is 3. (So x is either 5 or -1.) You can also use the the expressions above to solve this:

$$|x - 2| = 3 \Longrightarrow \begin{cases} x - 2 = 3 & \text{or } x = 5, \\ x - 2 = -3 & \text{or } x = -1 \end{cases}$$

b) With an inequality such as $|x - 2| < 3$, again use the basic definitions. $|x - 2| < 3$ means $-3 < (x - 2) < 3$ or $-1 < x < 5$.

6. The distance formula for the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This can be derived directly from the Pythagorean theorem if you draw the two points and a right triangle determined by them.

7. Equations of lines:

a) Slope-intercept form: $y = mx + b$.

b) Point-slope form: $y - y_0 = m(x - x_0)$, where (x_0, y_0) is a point on the line. This is particularly useful for calculus.

c) Know how to obtain the equation of line from two points (x_1, y_1) and (x_2, y_2).

8. Functions including domain and range. Composition of functions $(f \circ g)(x) = f(g(x))$. For example, if $f(x) = x^2 - 6$ and $g(x) = 1 + 2x^3$, then

$$(f \circ g)(x) = f(g(x)) = f(1 + 2x^3) = (1 + 2x^3)^2 - 6 = 1 + 4x^3 + 4x^6 - 6 = 4x^3 + 4x^6 - 5.$$

9. Basic geometry formulas:

a) Triangles: Area $A = \frac{1}{2}bh$.

b) Rectangles: Area $A = lw$; Perimeter $P = 2l + 2w$.

c) Circles: Area $A = \pi r^2$; Circumference $C = 2\pi r$.

d) Spheres: Volume $V = \frac{4}{3}\pi r^3$; Surface Area $SA = 4\pi r^2$.

e) Cylinder: Volume $V = \pi r^2 h$,;Surface Area $SA = 2\pi r^2 + 2\pi rh$.

f) Cone: Volume $V = \frac{1}{3}\pi r^2 h$.

g) Rectangular box: Volume::$V = lwh$; Surface Area $SA = 2lw + 2lh + 2wh$.

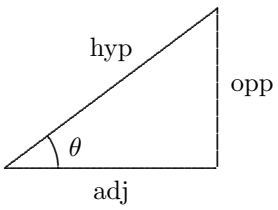
10. We will always measure angles in *radians*. The conversion factors are:

a) π radians $= 180°$.

b) So $1° = \frac{\pi}{180}$ rad

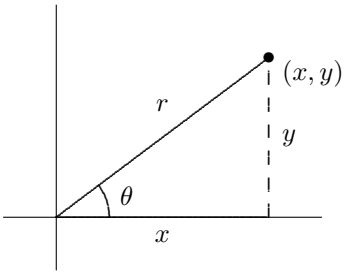
c) and 1 rad $= \frac{180}{\pi}°$.

11. Recall for a right triangle like the one below, we can define the basic trig functions in terms of the sides of the triangle.



a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ **b)** $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ **c)** $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ **d)** $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

12. For more general angles,



a) $\sin\theta = \dfrac{y}{r}$ **b)** $\cos\theta = \dfrac{x}{r}$ **c)** $\tan\theta = \dfrac{y}{x}$ **d)** $\sec\theta = \dfrac{r}{x}$

13. You **must** know the values of the trig functions at these basic angles. Memorize!

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–

Elementary Practice

Here's a quick review of even **more basic topics**. I assume that you know all of this material extremely well.

Exponents and radicals:

1. Simplify and evaluate: $(2\sqrt{5})(2\sqrt{5})(5\sqrt{2})(5\sqrt{2})$

2. Write in simplified exponential form:

a) $(x^3)^2$ **b)** $(x^3 \cdot y^2)^{-1}$ **c)** $(-2x)^{-5}$

d) $\dfrac{x \cdot y^{-2}}{y^{-4}x^{-1}}$ **e)** $(-2)^0$ **f)** $\left(z^{-2}\right)^{-2}$

3. Simplify each expression:

a) $\left(-8z^6\right)^{2/3}$ **b)** $6y^2\left(2y^4\right)^2$ **c)** $\dfrac{25y^8}{10y^4}$ **d)** $\dfrac{12(x+y)^3}{9(x+y)^5}$ **e)** $\dfrac{10(x-y)^{5/3}}{15(x-y)^{9/3}}$

4. Rewrite each expression below in simplified exponential form (no radicals):

a) $\dfrac{1}{\sqrt{x}}$ **b)** $\sqrt[9]{x^6}$ **c)** $6\sqrt[5]{y^2}$ **d)** $2\sqrt[9]{z^3}$ **e)** $\dfrac{1}{2\sqrt[4]{z^3}}$ **f)** $\sqrt[6]{(x+1)^4}$

5. Expand each of these powers:

a) $(x-2)^3$ **b)** $\left(x-\sqrt{5}\right)^2$

6. Factor each expression completely:

a) $x^2+10x+25$ **b)** $4x^2-4xy+y^2$ **c)** t^2-t-6 **d)** $49-9y^2$

e) $9u^2-4v^2$ **f)** $6x^2-54$ **g)** $-2x^3+2x^2+4x$

7. Simplify these rational functions.

a) $\dfrac{8(x+1)^4+7x^3(x+1)^2}{(x+1)^4}$ **b)** $\dfrac{9(x-2)^3(x+1)-2(x-2)^4(x-1)^2}{(x-2)^6}$

Composition

8. Let $f(x)=x^2-4x+1$, $g(x)=2-x$, and $h(x)=3\sin x$, determine the expressions for

a) $f\circ g(x)$ **b)** $h\circ g(x)$

Elementary Practice Answers

Exponents and radicals:

1. $(2\sqrt{5})(2\sqrt{5})(5\sqrt{2})(5\sqrt{2}) = (4 \times 5)(25 \times 2) = 1000$

2. Simplified expressions:

a) x^6 **b)** $x^{-3}y^{-2}$ **c)** $-32x^{-5}$
d) x^2y^2 **e)** 1 **f)** z^4

3. Simplified:

a) $4z^4$ **b)** $24y^{10}$ **c)** $\dfrac{5y^4}{2}$ **d)** $\dfrac{4(x+y)^{-2}}{3}$ **e)** $\dfrac{2(x-y)^{-4/3}}{3}$

4. Exponential form:

a) $\dfrac{1}{\sqrt{x}} = x^{-1/2}$ **b)** $\sqrt[9]{x^6} = x^{2/3}$ **c)** $6\sqrt[5]{y^2} = 6y^{2/5}$
d) $2\sqrt[9]{z^3} = 2z^{1/3}$ **e)** $\dfrac{1}{2\sqrt[4]{z^3}} = \dfrac{z^{-4/3}}{2}$ **f)** $((x+1)^4)^{1/6} = (x+1)^{2/3}$

5. Expanded powers:

a) $x^3 - 6x^2 + 12x - 8$ **b)** $x^2 - 2x\sqrt{5} + 5$

6. Factored:

a) $(x+5)^2$ **b)** $(2x-y)^2$ **c)** $(t-3)(t+2)$ **d)** $(7-3y)(7+3y)$
e) $(3u-2v)(3u+2v)$ **f)** $6(x-3)(x+3)$ **g)** $-2x(x-2)(x+1)$

7. Simplified:

a) $\dfrac{8(x+1)^2 + 7x^3}{(x+1)^2}$ **b)** $\dfrac{9(x+1) - 2(x-2)(x-1)^2}{(x-2)^3}$

8. Composition: Let $f(x) = x^2 - 4x + 1$, $g(x) = 2 - x$, and $h(x) = 3\sin x$, determine the expressions for

a) $f(g(x)) = f(2-x) = (2-x)^2 - 4(2-x) + 1 = x^2 - 3$
b) $h(g(x)) = h(2-x) = 3\sin(2-x)$

Math 130 Prereqs Hand In. Name: _____

Review the material on the previous pages as needed. You may wish to consult Appendix A and Chapter 1 of your text.

1. Simplify and factor completely: $(x+3)^2 - 4(x+3)$.

2. Notation and Piecewise Functions. (See pages 14–15, Examples 3 and 4.) Define

$$f(x) = \begin{cases} x+2 & \text{if } x < 1, \\ 2x-4 & \text{if } x \geq 1. \end{cases}$$

Evaluate the following:

a) $f(3) =$

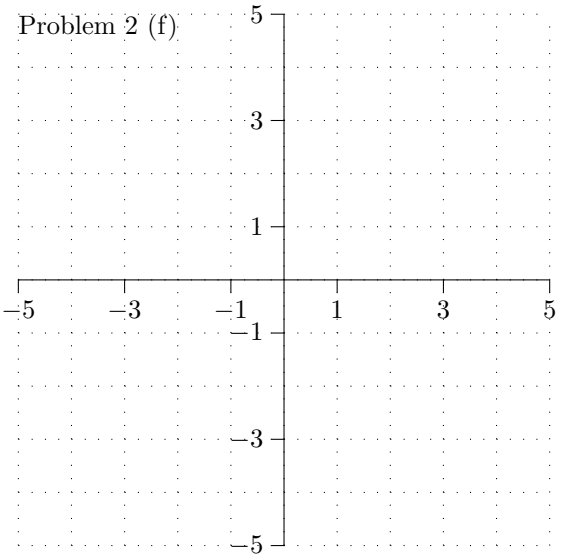
b) $f(-6) =$

c) $f(1) =$

d) $f(0.99) =$

e) $f(f(0)) =$

f) Draw a graph of $f(x)$ on the axes below.



The next problems are in the text. **Show your work.**

3. Page 10 #32 (see Page 6, Example 9)

4. Page 11 #80. (See page 8 Example 11. Try #79 that has answers in the back.)

A)

B)

C)

5. Page 22 #20. Your answer should be similar in form to the function in Problem 2 of this assignment.

$$f(x) = \begin{cases} \\ \\ \end{cases}$$

Work, if needed.

6. a) According to the bottom of page 32,

$$y = \log_b x \text{ if and only if } \_\_\_\_\_$$

b) Now do page 36 #42.

7. Give the exact value (no decimals) for each of these trig and inverse trig functions. (See pages 40 and 44.)

a) $\sin(\frac{\pi}{3}) =$

b) $\cos(\frac{\pi}{2}) =$

c) $\tan(\frac{\pi}{3}) =$

d) $\sin^{-1}(\frac{1}{2}) =$

e) $\cos^{-1}(\frac{1}{2}) =$

8. Complete the table, where each row describes the same interval. (See page 1152.)

Interval Notation	**Set Notation (Inequalities)**	**Number Line Graph of the Interval**
$(-\infty, -4]$		
		
	$\{x : \ 3 \leq x \leq \frac{11}{2}\}$	
$(-1, 7)$		

9. a) According to "Properties of the Absolute Value" in Appendix A page 1153: $|x| < a$ means the same thing as:

b) Use the property above to solve $|2x - 4| < 6$.

10. These are both optional **Extra Credit**.

a) Page 22 #38 (see Example 6 in the text).

b) Page 49 #98. You will need to use a calculator. Hint: Be sure to convert all measurements to feet! Let P denote the point where the line BA meets the x-axis (end line). Determine angle $\angle G_1AG_2$ as the difference between angles $\angle G_1AP$ and $\angle G_2AP$ by using the arctan function. Similarly for $\angle G_1BG_2$. What do you find? Give your answer in radians!