Math 131 PracTest 1

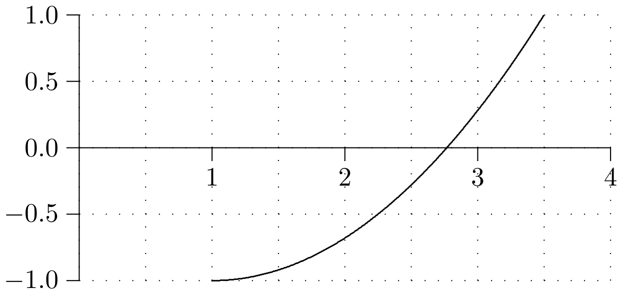
1. a) State the Mean Value Theorem and draw a graph that illustrates.

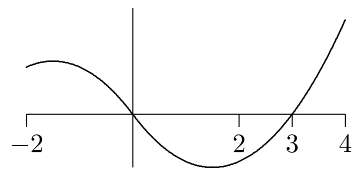
b) Name an important theorem where the Mean Value Theorem was used in the proof.

c) State the definition of the definite integral of a continuous function on a closed interval $[a, b]$.

d) Draw a picture that illustrates the Mean Value Theorem for Integrals.

2. Draw and then estimate R_5 for the graph of f on $[1, 3.5]$ on the left below. Be careful of the scale.





3. In the graph of f on the right above, assume that $\int_{-2}^{4} f(x)\,dx = 1$, $\int_{-2}^{0} f(x)\,dx = 3$, and $\int_{-2}^{3} f(x)\,dx = -1.2$. Evaluate the following.

a) $\displaystyle\int_3^4 f(x)\,dx$ **b)** $\displaystyle\int_0^3 f(x)\,dx$ **c)** $\displaystyle\int_{-2}^4 2f(x-1)\,dx$ **d)** $\displaystyle\int_{-2}^0 f(x) - 4\,dx$

e) If $f(x)$ is symmetric about the origin, what is $\int_0^2 f(x)\,dx$?

4. Calculate these "look-alike" indefinite integrals.

a) $\displaystyle\int \sqrt{4t-1}\,dt$ **b)** $\displaystyle\int \frac{1}{\sqrt{1-4t^2}}\,dt$ **c)** $\displaystyle\int \frac{t}{\sqrt{1-4t^2}}\,dt$ **d)** $\displaystyle\int t\sqrt{4-t}\,dt$

e) $\displaystyle\int \frac{t}{1+4t^4}\,dt$ **f)** $\displaystyle\int \frac{t^3}{1+4t^4}\,dt$ **g)** $\displaystyle\int \frac{t^2}{1+4t^6}\,dt$ **h)** $\displaystyle\int \frac{1+4t^6}{t^2}\,dt$

5. a) If an antiderivative of $f(x)$ is $\sin x - x\sin x + 1$, what was $f(x)$?

b) On a recent test Judy said that $\int 2\cos x \sin x\,dx = \sin^2 x + c$. Elaine said that $\int 2\cos x\sin x\,dx = -\cos^2 x + c$. I gave them both full-credit. How can both be correct if their answers are different? Did I make a mistake?

6. a) Fill in the table for R_n for $\int_1^3 3(x^2-1)\,dx$. Be sure to simplify $f(x_k)$.

$f(x)$	$[a, b]$	Δx	x_k	$f(x_k)$	R_n

b) If you were to graph $f(x)$ on $[1, 3]$ it would be an increasing function. Is R_n an over or under estimate of $\int_1^3 3x^2 - 3\,dx$? Explain.

c) Find $\int_1^3 3x^2 - 3\,dx$ using a limit of Riemann sums. Check your answer by evaluating $\int_1^3 3x^2 - 3\,dx$ using the FTC.

d) Think: On a test once, I asked students to compute R_{100} and R_{200} for $f(x) = e^x$ on an interval $[a, b]$. A student correctly computed both with his calculator but failed to label which is which. Which of her two values is R_{100}: 47.6828 or 47.4455? **Explain** clearly how you can tell.

7. Determine these antiderivatives

a) $\displaystyle\int 8\sec(4\pi x)\tan(4\pi x), dx$ **b)** $\displaystyle\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$ **c)** $\displaystyle\int \sin(\sin t)\cos t\,dt$ **d)** $\displaystyle\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

e) Evaluate $\displaystyle\int_0^1 \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$.

8. The graph of $f(x)$ is given below on the left. Answer the following questions about $F(x) = \int f(x)\,dx$.

a) On what interval(s) is F incresasing? Explain.

b) At what point(s), if any, does F have a local min? Explain.

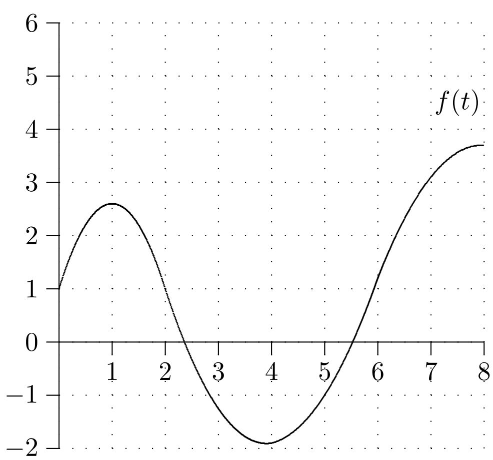
c) On what interval(s) is F concave down? Explain.

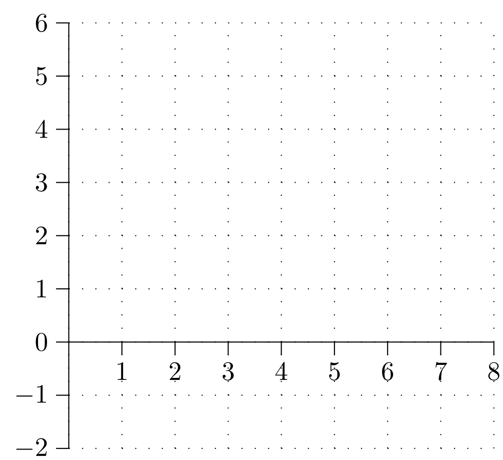
d) Does F have any points of inflection? Explain.

e) Fill in the the table below. Your values should be reasonably accurate.

b	0	1	2	3	4	5	6	7	8
$F(b) = \int_0^b f(t)\,dt$									

f) Graph $F(x)$ on the axes on the right using your table values.





9. Think and Interpret: The velocity of an object $v(t)$ in m/s is graphed above on the left (same graph as in previous question).

a) When is the object moving backwards? Explain.

b) When is acceleration negative? Explain.

c) Assume that the initial position of the object is $s(0) = 0$. Fill in the row in the table for the position $s(t)$ for the object. Then graph the position function on the axes above on the right.

t	0	1	2	3	4	5	6	7	8
Position $s(t)$									
Distance Traveled $= \int_a^b \lvert v(t)\rvert\,dt$									

d) What is v_{ave}? (Hint: Use the v_ave formula and your work above.) At what time(s) t did v_{ave} occur? Explain.

e) Speed is $|v(t)|$. Graph the speed function for the object on the axes on the left.

f) Distance traveled is the integral of the speed: $\int_a^b |v(t)|\, dt$. Fill in the row in the table above for the distance traveled by the object.

10. a) An oil supertanker traveling at 16 knots (1 knot is 1.6875 ft/s) requires 3 nautical miles (1 nautical mile is 6080 ft) to stop with the engines in full-reverse. Find the deceleration for the tanker, assuming this rate is constant. Hint: Convert all units to feet. [Ans: $a = -\frac{729}{36480}$ ft/s^2]

b) How much time passes during this process?

11. My Honda Accord accelerates from 0 to 88 ft/sec (60 mph) in 13 seconds. Assume that the acceleration is a constant, a.

a) Find the velocity function of the car (using the velocities at the two times you can eliminate any constants from this function).

b) How far does it travel in this 13 second period?

Math 131: Answers to Practest 1

1. a) Mean Value Theorem: See page 208.

b) FTC, Mean Value Theorem for Integrals.

c) Partition the interval $[a, b]$ into n equal width subintervals using points

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b.$$

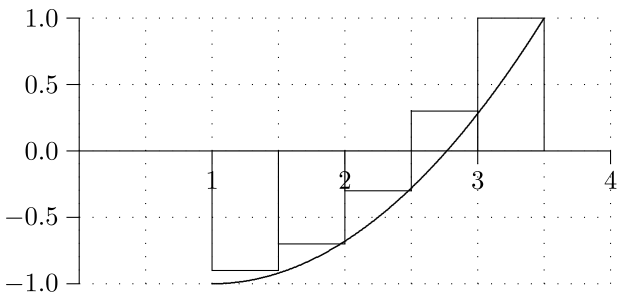
The definite integral of a continuous function f on a closed interval $[a, b]$ is

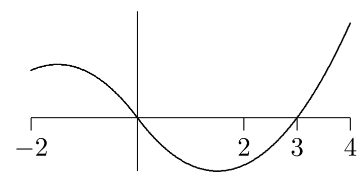
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and x_k^* is some point in the in k-th subinterval.

d) Draw a picture that illustrates the Mean Value Theorem for Integrals. See page 312.

2. $\Delta x = \frac{3.5-1}{5} = \frac{1}{2}$. So $R_5 = (-.9)\cdot\frac{1}{2} + (-.7)\cdot\frac{1}{2} + (-.3)\cdot\frac{1}{2} + (.3)\cdot\frac{1}{2} + (1)\cdot\frac{1}{2} = -.3$.





3. Use a basic integral properties.

a) $\int_3^4 f(x)\,dx = \int_{-2}^4 f(x)\,dx - \int_{-2}^3 f(x)\,dx = 1 - (-1.2) = 2.2$

b) $\int_0^3 f(x)\,dx = \int_{-2}^3 f(x)\,dx - \int_2^0 f(x)\,dx = -1.2 - 3 = -4.2$

c) $\int_{-1}^5 2f(x-1)\,dx = 2\int_{-2}^4 f(x)\,dx = 2 \quad \text{(horizontal shift)}$

d) $\int_{-2}^0 f(x) - 4\,dx = 3 + (-4)[0-(-2)] = -5$

e) $\int_0^2 f(x)\,dx = -\int_{-2}^0 f(x)\,dx = -3$

4. a) $u = 4t-1$, $\frac{1}{4}du = dt$. $\frac{1}{4}\int u^{1/2}\,du = \frac{1}{6}u^{3/2} + c = \frac{1}{6}(4t-1)^{3/2} + c$.

b) $u^2 = 4t^2$, $u = 2t$, $\frac{1}{2}du = dt$. $\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin u + c = \frac{1}{2}\arcsin(2t) + c$.

c) $u = 4t-1$, $\frac{1}{4}du = dt$. $\frac{1}{4}\int u^{1/2}\,du = \frac{1}{6}u^{3/2} + c = \frac{1}{6}(4t-1)^{3/2} + c$.

d) $u = 4-t$, $-du = dt$, $t = 4-u$. $-\int(4-u)u^{1/2}\,du = -\int 4u^{1/2} - u^{3/2}\,du = \frac{8}{3}u^{3/2} - \frac{2}{5}u^{5/2} + c = \frac{8}{3}(4-t)^{3/2} - \frac{2}{5}(4-t)^{5/2} + c$.

e) $u^2 = 4t^4$, $u = 2t^2$, $\frac{1}{4}du = tdt$. $\frac{1}{4}\int \frac{1}{1+u^2}\,du = \frac{1}{4}\arctan u + c = \frac{1}{4}\arctan(2t^2) + c$.

f) $u = 1+4t^4$, $\frac{1}{16}du = t^3dt$. $\frac{1}{16}\int \frac{1}{u}\,du = \frac{1}{16}\ln|u| + c = \frac{1}{16}\ln(1+4t^4) + c$.

g) $u^2 = 4t^6$, $u = 2t^3$, $\frac{1}{6}du = t^2dt$. $\frac{1}{6}\int \frac{1}{1+u^2}\,du = \frac{1}{6}\arctan u + c = \frac{1}{6}\arctan(2t^3) + c$.

h) $\int \frac{1+4t^6}{t^2}\,dt = \int t^{-2} + 4t^4\,dt = -t^{-1} + \frac{4}{5}t^5 + c$. (Just divide first by t^2.)

5. a) Since $\int f(x)\,dx = \sin x - x\sin x + 1$, then $f(x) = \frac{d}{dx}(\sin x - x\sin x + 1) = \cos x - \sin x - x\cos x$.

b) No. Just take the derivatives of each: $\frac{d}{dx}(\sin^2 x + c) = 2\sin x\cos x$ while $\frac{d}{dx}(-\cos^2 x + c) = -2\cos x(-\sin x) = 2\sin x\cos x$ also. The two antidervatives differ by a constant since $\sin^2 x = -\cos^2 x + 1$.

6. a)

$f(x)$	$[a,b]$	Δx	x_k	$f(x_k)$	R_n
$3(x^2-1)$	$[1,3]$	$\frac{2}{n}$	$1+\frac{2k}{n}$	$3\left(\frac{4k^2}{n^2}+\frac{4k}{n}\right)$	$\sum_{k=1}^{n} 3\left(\frac{4k^2}{n^2}+\frac{4k}{n}\right)\left(\frac{2}{n}\right)$

b) R_n is an overestimate of $\int_1^3 3x^2 - 3\,dx$ because $f(x_k)$ will be greater than any value of $f(x)$ in $[x_{k-1}, x_k]$ since the function is increasing.

c) $$R_n = 3\sum_{k=1}^{n}\left[\frac{4k^2}{n^2}+\frac{4k}{n}\right]\cdot\frac{2}{n} = \frac{24}{n^3}\sum_{k=1}^{n}k^2 + \frac{24}{n^2}\sum_{k=1}^{n}k = \frac{24}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{24}{n^2}\left[\frac{n(n+1)}{2}\right]$$

$$= 4\left[\frac{2n^2+3n+1}{n^2}\right] + \frac{12n+12}{n} = 8 + \frac{12}{n} + \frac{4}{n^2} + 12 + \frac{12}{n} = 20 + \frac{24}{n} + \frac{4}{n^2}$$

So $\int_1^3 3(x^2-1)\,dx = \lim_{n\to\infty} R_n = 20$ and using the FTC, $\int_1^3 3x^2 - 3\,dx = x^3 - 3x\big|_1^3 = (27-9) - (1-3) = 20$.

d) Since e^x is an increasing function R_n is an overestimate. Since the estimates improve (smaller overestimate) as n gets larger, then $R_{100} > R_{200}$. So $R_{100} = 47.6828$.

7. a) $\frac{8}{4\pi}\sec(4\pi x) + c = \frac{2}{\pi}\sec(4\pi x) + c$. (Mental adustment)

b) $u = \sqrt{t}$, $2du = \frac{1}{\sqrt{t}}dt$. $2\int e^u\,du = 2\ln e^u + c = 2e^{\sqrt{t}} + c$.

c) $u = \sin t$, $du = \cos t dt$. $\int \sin u\,du = -\cos(u) + c = -\cos(\sin t) + c$.

d) $u = e^x - e^{-x}$, $du = (e^x + e^{-x})dx$. $\int \frac{1}{u}\,du = \ln|u| + c = \ln|e^x - e^{-x}| + c$.

8. The graph of $f(x)$ is given below on the left. Answer the following questions about $F(x) = \int f(x)\,dx$.

a) F incresasing, so $F' = f > 0$: $[0, 2.5]$ and $[5.5, 8]$.

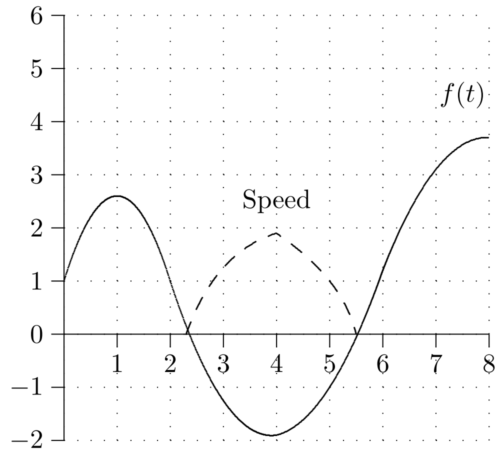
b) Local min when $F' = f$ changes from negative to positive: $x = 5.5$

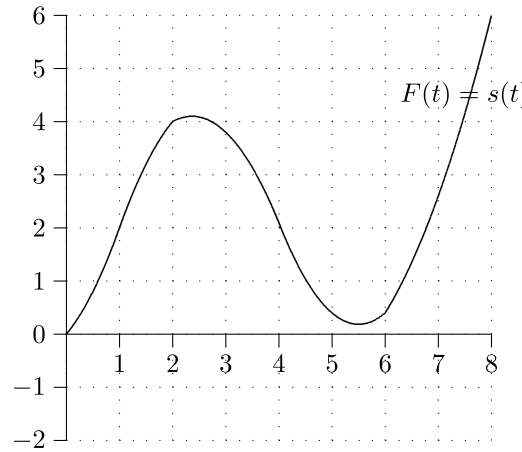
c) F concave down when $F'' = f' < 0$, i.e., when f is decreasing: $[1, 4]$.

d) Points of inflection where $F'' = f'$ changes sign, i.e., where f' changes from increasing to decreasing or vice versa: $x = 1$ and $x = 4$.

e) $\int_0^b f(t)\,dt$ is just the net area under the curve from 0 to b.

b	0	1	2	3	4	5	6	7	8
$F(b) = \int_0^b f(t)\,dt$	0	2	4	3.8	2.1	0.4	0.4	2.6	6





9. a) When is the object moving backwards? $v < 0$ so $[2, 5, 5.5]$.

b) Negative acceleration when $a = v' < 0$: so $[1, 4]$.

c) and **(f)** Position is the net area under the curve (same table as above).

t	0	1	2	3	4	5	6	7	8
Position $s(t)$	0	2	4	3.8	2.1	0.4	0.4	2.6	6
Distance Traveled	0	2	4	4.8	6.5	8.2	9	11.2	14.6

d) $v_{ave} = \frac{1}{8-0}\int_0^8 v(t)\,dt = \frac{1}{8}[s(8) - s(0)] = \frac{1}{8}(6-0) = \frac{3}{4}$. v_{ave} occurred at $c = 2.2$ and 5.8. [Use the graph of $v(t) = f(t)$.]

10. Given: $a(t) = a$ constant. $v_0 = 27$ ft/s (16 knots). $s_0 = 0$. We need to find the time t^* so that $v(t^*) = 0$ when $s(t^*) = 18240$ ft (3 nautical miles). Now $v(t) = at + v_0 = at + 27$. When $v(t) = at + 27 = 0$, then $t = -\frac{27}{a}$. This is the time at which the position is 18240 ft.

a) $s(t) = \frac{1}{2}at^2 + v_0t + s_0 = \frac{1}{2}at^2 + 27t$. So at time $t = -\frac{27}{a}$, the position is 18240 ft or:

$$s(-\tfrac{27}{a}) = \tfrac{1}{2}a(-\tfrac{27}{a})^2 + 27(-\tfrac{27}{a}) = \tfrac{729}{2a} - \tfrac{729}{a} = -\tfrac{729}{2a} = 18240 \Rightarrow a = -\tfrac{729}{36480} = -0.0199835526\ \text{ft/s}^2.$$

b) The time to stop is $t = -\frac{27}{a} = 1351.\overline{1}$ seconds or 22.52 minutes.

11. Given: $a(t) = a$ constant. $v_0 = 88$ ft/s. $s_0 = 0$. And $v(13) = 0$ ft/s.

a) $v(t) = at + v_0 = at + 88$. But $v(13) = 13a + 88 = 0 \Rightarrow a = -\frac{88}{13}$. So $v(t) = -\frac{88}{13}t + 88$.

b) $s(t) = -\frac{1}{2}at^2 + v_0t + s_0 = -\frac{44}{13}t^2 + 88t$.

c) $s(13) = -\frac{44}{13}(13)^2 + 88(13) = 572$ ft.