Hand in next class. These are (almost) the same as the WeBWorK Day 04 assignment. Do them together; check your answers!

1. (*a*) Fill in the following table for the Riemann sum using regular partitions and right-hand endpoints.

$f(x)$	$[a,b]$	Δx	$x_k = a + k\Delta x$	$f(x_k)$	Write out Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$ (Do not simplify yet)
$(x-1)^2$	$[0,2]$	$2/n$	$0 + 2k/n = 2k/n$	$(\frac{2k}{n}-1)^2$	$\sum_{k=1}^{n} (\frac{2k}{n}-1)^2\, 2/n$

(*b*) Simplify Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$. No sum should appear.

$$\sum_{k=1}^{n} \left(\frac{2k}{n}-1\right)^2\left(\frac{2}{n}\right) = \sum_{k=1}^{n}\left(\frac{4k^2}{n^2} - \frac{4k}{n} + 1\right)\frac{2}{n} = \frac{8}{n^3}\sum_{k=1}^{n} k^2 - \frac{8}{n^2}\sum_{k=1}^{n} k + \frac{2}{n}\sum_{k=1}^{n} 1$$

$$= \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{8}{n^2}\,\frac{n(n+1)}{2} + \frac{2}{n}(n\cdot 1)$$

$$= \frac{4(2n^2+3n+1)}{3n^2} - \frac{4(n+1)}{n} + 2 = \frac{8}{3} + \cancel{\frac{4}{n}} + \frac{4}{3n^2} - 4 - \cancel{\frac{4}{n}} + 2$$

$$= 2/3 + \frac{4}{3n^2}$$

~~(*c*) Is Right(n) an over or under estimate? How can you tell?~~

(*d*) $\lim_{n\to\infty}$ Right$(n) = \lim_{n\to\infty} \frac{2}{3} + \frac{4}{3n^2} = 2/3$

(*e*) When $f(x)$ is continuous, this limit is denoted by $\int_a^b f(x)\,dx$. Here we would write $\int_0^2 (x-1)^2\,dx = \lim_{n\to\infty}$ Right$(n) =$ $2/3$

2. (*a*) Fill in the following table for the Riemann sum using regular partitions and right-hand endpoints.

$f(x)$	$[a,b]$	Δx	$x_k = a + k\Delta x$	$f(x_k)$	Write out Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$ (Do not simplify yet)
$4(x-2)^3$	$[2,4]$	$\frac{2}{n}$	$2+\frac{2k}{n}$	$4(\frac{2k}{n})^3$	$\sum_{k=1}^{n} 4(\frac{2k}{n})^3(\frac{2}{n})$

(*b*) Simplify Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$. No sum should appear. This simplifies a lot!

$$\sum_{k=1}^{n} 4\left(\frac{2k}{n}\right)^3\left(\frac{2}{n}\right) = \frac{64}{n^4}\sum_{k=1}^{n} k^3 = \frac{64}{n^4}\,\frac{n^2(n+1)^2}{4} = \frac{16\,(n+1)^2}{n^2}$$

$$= 16\left(\frac{n^2+2n+1}{n^2}\right) = 16 + \frac{32}{n} + \frac{16}{n}$$

~~(*c*) Is Right(n) an over or under estimate? How can you tell?~~

(*d*) [Hint: See Problem 1(d,e).] Here $f(x)$ is continuous, so $\int_2^4 4(x-2)^3\,dx = \lim_{n\to\infty}$ Right$(n) = \lim_{n\to\infty} 16 + \frac{32}{n} + \frac{16}{n} = 16$

3. (a) Fill in the following table for the Riemann sum using regular partitions and right-hand endpoints.

$f(x)$	$[a,b]$	Δx	$x_k = a + k\Delta x$	$f(x_k)$	Write out Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$ (Do not simplify yet)
$-4x^2$	$[0,2]$	$\frac{2}{n}$	$\frac{2k}{n}$	$-4\left(\frac{2k}{n}\right)^2$	$\sum -4\left(\frac{2k}{n}\right)^2\left(\frac{2}{n}\right)$

(b) Simplify Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$. No sum should appear.

$$\sum_{k=1}^{n} -4\left(\frac{2k}{n}\right)^2\left(\frac{2}{n}\right) = -\frac{32}{n^3}\sum_{k=1}^{n} k^2 = \frac{-32}{n^3}\,\frac{n(n+1)(2n+1)}{6} = -\frac{16(n+1)(2n+1)}{3n^2}$$

$$= -16\,\frac{(2n^2+3n+1)}{3n^2} = \frac{-32}{3} - \frac{16}{n} - \frac{16}{3n^2}$$

~~(c) Is Right(n) an over or under estimate? How can you tell?~~

(d) [Hint: See Problem 1(d,e).] Here $f(x)$ is continuous, so $\int_0^2 -4x^2\,dx = \lim_{n\to\infty} -\frac{32}{3} - \frac{16}{n} - \frac{16}{3n^2} = -\frac{32}{3}$

4. (a) Fill in the following table for the Riemann sum using regular partitions and right-hand endpoints.

$f(x)$	$[a,b]$	Δx	$x_k = a + k\Delta x$	$f(x_k)$	Write out Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$ (Do not simplify yet)
$2(x-1)^2$	$[1,4]$	$\frac{3}{n}$	$1+\frac{3k}{n}$	$2\left(\frac{3k}{n}\right)^2$	$\sum_{k=1}^{n} 2\left(\frac{3k}{n}\right)^2\left(3/n\right)$

(b) Simplify Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$. No sum should appear. This simplifies a lot!

$$\sum_{k=1}^{n} 2\left(\frac{3k}{n}\right)^2\left(\frac{3}{n}\right) = \frac{54}{n^3}\sum_{k=1}^{n} k^2 = \frac{54}{n^3}\,\frac{n(n+1)(2n+1)}{6} = \frac{9(2n^2+3n+1)}{n^2}$$

$$= 18 + \frac{27}{n} + \frac{9}{n^2}$$

~~(c) Is Right(n) an over or under estimate? How can you tell?~~

(d) [Hint: See Problem 1(d,e).] Here $f(x)$ is continuous, so $\int_1^4 2(x-1)^2\,dx = \lim_{n\to\infty} 18 + \frac{27}{n} + \frac{9}{n^2} = 18$