Math 131 Day 5

#1 $\int_2^3 (x^2-4)\,dx$

Use Right(n): $\Delta x = \frac{b-a}{n} = \frac{3-2}{n} = \frac{1}{n}$

Right endpt $\longrightarrow x_k = a + k\Delta x = 2 + \frac{k}{n}$

$f(x_k) = f(2+\frac{k}{n}) = 4 + \frac{4k}{n} + \frac{k^2}{n^2} - 4 = \frac{4k}{n} + \frac{k^2}{n^2}$

$$\text{Right}(n) = \sum_{k=1}^{n} \left(\frac{4k}{n} + \frac{k^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{4}{n^2}\sum_{k=1}^{n} k + \frac{1}{n^3}\sum_{k=1}^{n} k^2$$

5 $$= \frac{4}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{2n+2}{n} + \frac{2n^2+3n+1}{6n^2} = 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

So $\int_2^3 (x^2-4)\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} 7/3 + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2} = \boxed{\frac{7}{3}}$

#2 a) $f(x) = x^2 + x$ on $[0,2]$ $f'(x) = 2x+2 > 0$ on $[0,2]$
so f is increasing on $[0,2]$. So Upper(n) = Right(n)
6 and Lower(n) = Left(n) since f is increasing

b) Upper(n) = Right(n). $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

right endpt $\longrightarrow x_k = a + k\Delta x = 0 + \frac{2k}{n} = \frac{2k}{n}$

$f(x_k) = \left(\frac{2k}{n}\right)^2 + \left(\frac{2k}{n}\right) = \frac{4k^2}{n^2} + \frac{2k}{n}$

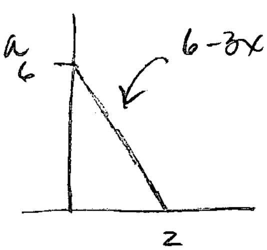
$$\text{Upper}(n) = \sum_{k=1}^{n}\left(\frac{4k^2}{n^2} + \frac{2k}{n}\right)\left(\frac{2}{n}\right) = \frac{8}{n^3}\sum_{k=1}^{n} k^2 + \frac{4}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{4}{n^2}\,\frac{n(n+1)}{2}$$

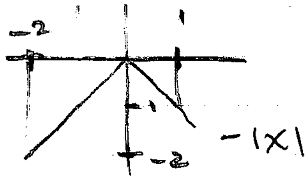
$$= \frac{4(2n^2+3n+1)}{3n^2} + \frac{2(n+1)}{n} = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n}$$

$$\int_0^2 x^2+2x\,dx = \lim_{n\to\infty} \text{Upper}(n) = \lim_{n\to\infty} \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n} = \boxed{\frac{14}{3}}$$

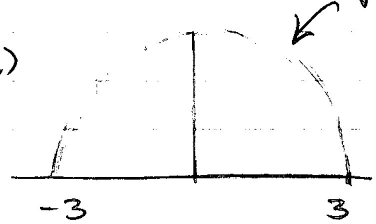
#3 a



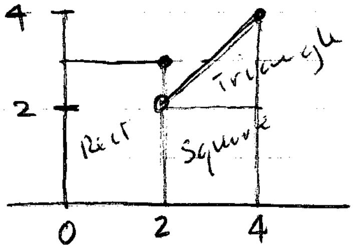
$\int_0^2 6-3x\,dx$ = Area of triangle $= \frac{1}{2}(2)(6) = 6$

#3b) 

$\int_{-2}^{1} -|x|\,dx =$ net area of triangles
$= \frac{1}{2}(2)(-2) + \frac{1}{2}(1)(-1) = -2\frac{1}{2}$

c) 

$\int_{-3}^{3} \sqrt{9-x^2}\,dx =$ net area of semi-circle
$= \frac{9}{2}\pi$

d) 

$\int_0^4 f(x)\,dx =$ net area

Rect, Square, Triangle

$= 2(3) + 2(2) + \frac{1}{2}(2)(2)$
$= 12$

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#4 a) #38 $\int_0^{3\pi/2} x\sin x\,dx = \int_0^{\pi/2} x\sin x\,dx + \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\frac{\pi}{2}} x\sin x\,dx$

$= 1 + (\pi - 1) - (\pi + 1)$

2 $= -1$ — below axis ... negative area

#40 $\int_{\pi/2}^{2\pi} x\sin x\,dx = \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx + \int_{3\pi/2}^{2\pi} x\sin x\,dx$

below x-axis

2 $= (\pi - 1) - (\pi + 1) - (2\pi - 1)$
$= -2\pi - 1$

b) #42 $\rightarrow \int_1^4 -3f(x)\,dx = -3\int_1^4 f(x)\,dx = -3(8) = -24$

2 c) $\int_6^4 12 f(x)\,dx = -12\int_4^6 f(x)\,dx$

$= -12\left[\int_1^6 f(x)\,dx - \int_1^4 f(x)\,dx\right]$

$= -12[5-8] = \boxed{36}$

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c) #44 a) $\int_0^{\pi/2} (2\sin\theta - \cos\theta)\,d\theta = -1$ (?)

(4) b) $\int_{\pi/2}^{0} (4\cos\theta - 8\sin\theta)\,d\theta = -\int_0^{\pi/2} 4\cos\theta - 8\sin\theta\,d\theta = -(-4)\int_0^{\pi/2} 2\sin\theta - \cos\theta\,d\theta$
$= -(-4)(-1) = -4$

#5 a) $\int \sqrt{7x}\,dx = \frac{2}{3}\frac{(7x)^{3/2}}{7} + C = \frac{2}{21}(7x)^{3/2} + C$ | (b) $\int \sin\frac{\pi x}{3}\,dx$
$= -\frac{3}{\pi}\cos\frac{\pi x}{3} + C$

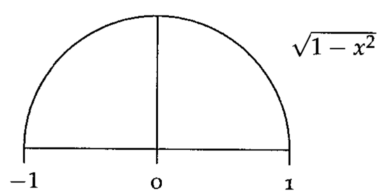
2 or $\int \sqrt{7}\,x^{1/2}\,dx = \frac{2\sqrt{7}}{3} x^{3/2} + C$ 2

6. (*a*) What is the area of a semi-circle of radius 1? Give your answer in terms of π and also as a decimal rounded correctly to four places. $1/2\,\pi \approx 1.5708$

(*b*) The rest of this problem uses the online calculator at `https://www.desmos.com/calculator/tgyr42ezjq` The equation of the semi-circle of radius 1 centered at the origin is $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$ (see figure). We should be able to find the area of this region using calculus. According to our theory, since $f(x) = \sqrt{1-x^2}$ is continuous, it is integrable so

$$\text{Area} = \int_{-1}^{1} \sqrt{1-x^2}\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \text{Left}(n).$$

So we should be able to approximate the answer using left and right Riemann sums with increasingly large values of n. Use the calculator at the website above (there's a link at our webpage) to find: Left(5), Right(5), and Midpoint(5). You will have to use the "choice of method" directions at the website. Then determine Left(50), Right(50), and Midpoint(50). Finally Left(100), Right(100), and Midpoint(100). Correctly round to four decimal places. Note: $\sqrt{1-x^2}$ is typed as `sqrt(1-x^2)`. The image below shows the values for 10 subintervals.



n	Left(n)	Right(n)	Midpoint(n)
5	1.4238	1.4238	1.6132
50	1.5661	1.5661	1.5722
100	1.5691	1.5691	1.5713

(*c*) Are these estimates getting closer to your answer in part (a) as n gets larger?