My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

Practice. Read 5.4 on average values and the Mean Value Theorem for Integrals. Review 5.3 as needed.

1. (*a*) Practice is important. Page 373ff. Try #9, 11, 13 and 15.
   (*b*) Using FTC I: Page 374 #61–6 and 101. (Even Answers: e^x, $-\frac{2x}{x^2+1}$, $-\frac{1}{x^2+1}$.)
   (*c*) Working with definite integrals: Page 376 #87 (simplify first), 89, and 91.
   (*d*) Assigned last time: Working with definite integrals: Page 374: #23, 27, 33, 37–43(odd), and 57. Remember, *net area* is signed area, so area below the axis is negative.

Hand In Due Next Time

0. WeBWorK set `Day07` (due Thursday night). Some of the Hand-in problems are similar. Do them together.

1. Review: This problem asks you to compute a definite integral two different ways: using Riemann sums and using the FTC. Review the Homework I handed back. The answers are on line.
   (*a*) Determine and simplify the formula for Right(n) for the function $f(x) = x^2 - x$ on the interval $[1, 4]$. Do this on another sheet and staple it to this one. Put your final simplified formula below: $\Delta x = \frac{3}{n}$ $x_k = 1 + \frac{3k}{n}$

$$\text{Right}(n) = \sum_{k=1}^{n}\left[\left(1+\frac{3k}{n}\right)^2 - \left(1+\frac{3k}{n}\right)\right]\frac{3}{n} = \sum_{k=1}^{n}\left[1+\frac{6k}{n}+\frac{9k^2}{n^2}-1-\frac{3k}{n}\right]\frac{3}{n}$$

5 $$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}+\frac{9k^2}{n^2}\right) = \frac{9}{n^2}\sum_{k=1}^{n}k + \frac{27}{n^3}\sum_{k=1}^{n}k^3 = \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{9}{2}\left(\frac{n+1}{n}\right) + \frac{9}{2}\left(\frac{2n^2+3n+1}{n^2}\right) = \frac{9}{2} + \frac{9}{2n} + 9 + \frac{27}{2n} + \frac{9}{2n^2}$$

   (*b*) Determine the value of $\int_1^4 (x^2 - x)\,dx$ by using a limit of Riemann sums. Use correct limit notation.

2 $$\int_1^4 (x^2-x)\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \frac{9}{2} + \frac{9}{2n} + 9 + \frac{27}{2n} + \frac{9}{2n^2} = \frac{9}{2} + 9$$
$$= \boxed{13\tfrac{1}{2}}$$

   (*c*) Using the Fundamental Theorem of Calculus, quickly evaluate $\int_1^4 (x^2 - x)\,dx$. (Are the answers the same?)

2 $$\int_1^4 x^2 - x\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_1^4 = \left[\frac{64}{3} - 8\right] - \left[\frac{1}{3} - \frac{1}{2}\right] = 21 - 7\tfrac{1}{2}$$
$$= \boxed{13\tfrac{1}{2}}$$

2. (*a*) Page 359 #38. Be careful, net area is signed area. Show your work using properties of the integral.

under x-axis

2 $$\int_0^{3\pi/2} x\sin x\,dx = \int_0^{\pi/2} x\sin x\,dx + \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx = 1 + (\pi - 1) - (\pi + 1)$$
$$= -1$$

   (*b*) Use the diagram on page 359 for #35–38 to determine $\int_{2\pi}^{0} x\sin x\,dx$. Be careful of signs. Show your work using properties of the integral.

2 $$\int_{2\pi}^{0} x\sin x\,dx = -\int_0^{2\pi} x\sin x\,dx = -\left[\int_0^{\pi/2} x\sin x\,dx + \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx + \int_{3\pi/2}^{2\pi} x\sin x\,dx\right]$$
$$= -\left[1 + (\pi - 1) - (\pi + 1) - (2\pi - 1)\right] = \boxed{2\pi}$$

3. Use the FTC (which part) to evaluate the following. Show your work.

4 (a) $\int_1^2 \left(\frac{2}{s} - \frac{4}{s^2}\right) ds =$ $2\ln|s| + \frac{4}{s} \Big|_1^2 = (2\ln 2 + 2) - (0 + 4) = 2\ln 2 - 2$

$- 4s^{-2}$

4 (b) $\int_0^{2\pi} \sec\frac{x}{8} \tan\frac{x}{8}\, dx =$ $8\sec\frac{x}{8}\Big|_0^{2\pi} = 8\sec\frac{2\pi}{8} - 8\sec 0 = 8\sec\frac{\pi}{4} - 8(1)$

$= 8\sqrt{2} - 8$

4. Use the FTC (which part) to simplify the following. Show your work. (See Example 5, p. 369.)

2 (a) $\frac{d}{dx}\left[\int_3^x t^2 \ln t\, dt\right] =$ $x^2 \ln x$

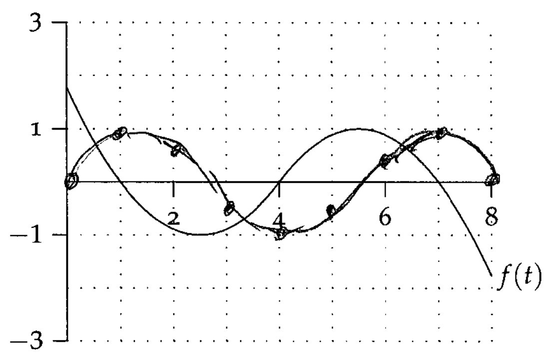
2 (b) $\frac{d}{dx}\left[\int_x^{12} \cos(t^3)\, dt\right] =$ $\frac{d}{dx}\left[-\int_{12}^{x} \cos(t^3)\, dt\right] = -\cos(x^3)$

$u = \sin x, \quad du/dx = \cos x$

2 (c) $\frac{d}{dx}\left[\int_0^{\sin x} \frac{1}{1+t^6}\, dt\right] =$ $f(u)\frac{du}{dx} = \frac{1}{1+u^6} \cdot \frac{du}{dx} = \frac{\cos x}{1 + \sin^6 x}$

$u = \sin x$

5. This is just like the earlier graphing problems you did on Lab. Review if necessary. Let $A(x) = \int_0^x f(t)\, dt$, where $f(t)$ is the function graphed below. $A(x)$ **is the** ***net area*** **between** f **and the axis on the interval between** 0 **and the endpoint** x. Use this relationship and the part of the Fundamental Theorem that we proved today in class to answer the following questions. First determine:



(a) $A(0) = 0$ $A(1) = 1$ $A(2) = \frac{1}{2}$ $A(3) = -1/2$

$A(4) = -1$ $A(5) = -1/2$ $A(6) = 1/2$ $A(7) = 1$ $A(8) = 0$

(b) On what interval(s) is A increasing? Explain briefly.

Where $f(x) = A'(x) > 0$ on $(0,1) \cup (4,7)$

(c) At what point(s), if any, does A have a local max?

Yes $x = 1$ and 7 $A' = f$ switches from $+$ to $-$ 2

What about mins?

$x = 4$ 1

(d) Make a rough sketch of the graph of $A(x)$ on the same axes using your values of A including maxs and mins.

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