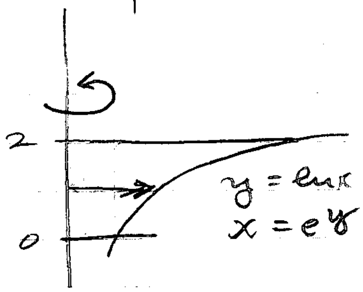
Math 131 Day 15

#5 p432 #36

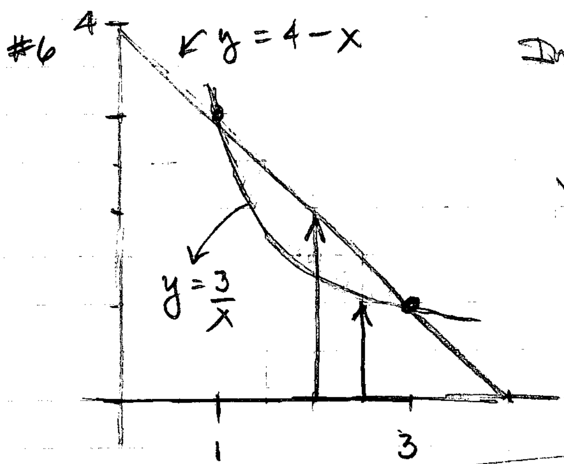


Cross-Area $= A(y) = \pi(e^y)^2 = \pi e^{2y}$

$$V = \int_0^2 A(y)\,dy = \int_0^2 \pi e^{2y}\,dy$$

$$= \frac{\pi}{2} e^{2y}\Big|_0^2 = \frac{\pi}{2}\left[e^4 - 1\right] \text{ units}^3$$

#6



Intersect: $\frac{3}{x} = 4 - x \Rightarrow 3 = 4x - x^2$

$x^2 - 4x + 3 \quad (x-1)(x-3) = 0 \quad x = 1, 3$

V = Outer − Inner

$$= \int_1^3 \pi\,(4-x)^2\,dx - \int_1^3 \pi\left(\tfrac{3}{x}\right)^2 dx$$

$$= \pi\int_1^3 16 - 8x + x^2\,dx - \pi\int_1^3 9x^{-2}\,dx$$

Better $u = 4 - x$
$du = -dx$
$-du = dx$

$$= \pi\left(16x - 4x^2 + \frac{x^3}{3}\right)\Big|_1^3 - \pi\left(-9x^{-1}\right)\Big|_1^3$$

$-\frac{(4-x)^3}{3}$ etc

$$= \pi\left(48 - 36 + 9\right) - \left(16 - 4 + \tfrac{1}{3}\right)\Big] - \pi\left[-3 - (-9)\right]$$

$$= 8\tfrac{2}{3}\pi - 6\pi = \frac{8}{3}\pi \text{ units}^3$$

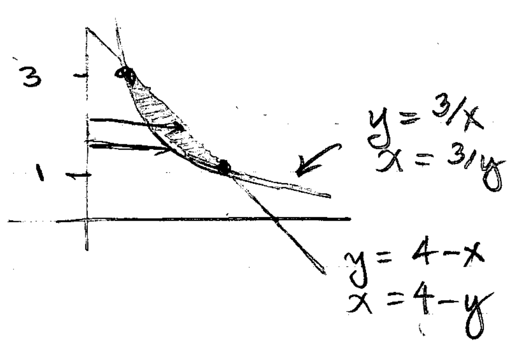
Around y-axis: $y = 3/x \Rightarrow x = 3/y$

$y = 4 - x \Rightarrow x = 4 - y$

Intersections: when $x = 1 \Rightarrow y = 3$; when $x = 3 \Rightarrow y = 1$

$$V = \text{Outer} - \text{Inner} = \int_1^3 \pi(4-y)^2\,dy - \int_1^3 \pi\left(\tfrac{3}{y}\right)^2 dy$$

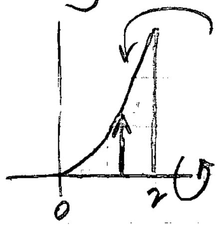
SAME AS ABOVE!



$= \frac{8\pi}{3}$ units^3

Day 15 Math 131

#1



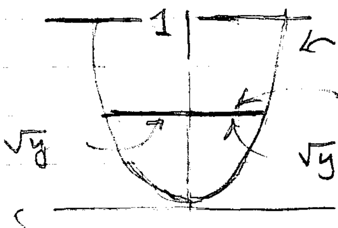
$y = x^3 =$ radius

$$V = \int_a^b \pi (f(x))^2 \, dx = \pi \int_0^2 (x^3)^2 \, dx = \pi \int_0^2 x^6 \, dx$$

$$= \frac{\pi}{7} x^7 \Big|_0^2 = \boxed{\frac{128\pi}{7}} \text{ units}^3$$

5

#2 p430 #12



$A(y) =$ square $= (\text{edge})^2 = (2\sqrt{y})^2 = 4y$

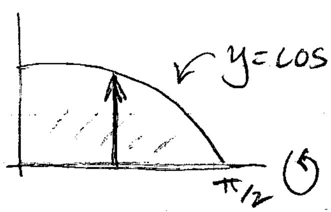
$$V = \int_0^1 A(y)\, dy = \int_0^1 4y \, dy$$

$$= 2y^2 \Big|_0^1 = 2 \text{ units}^3$$

5

#3 p431 #20

6



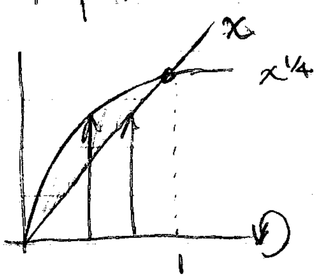
$A(x) =$ circle $= \pi r^2 = \pi (\cos x)^2$

$$V = \int_0^{\pi/2} \pi (\cos x)^2 dx = \pi \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2x)\, dx$$

$$= \frac{\pi}{2} \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\pi/2}$$

$$= \frac{\pi}{2} \left(\left(\frac{\pi}{2} + 0\right) - 0 \right) = \pi^2/4 \text{ units}^3$$

#4 p 431 #28



Intersect: $x^{1/4} = x \Rightarrow x = x^4 \Rightarrow x^4 - x = 0$

So $x(x^3 - 1) = 0$; $x = 0$ or $x = 1$

Vol = Outer − Inner

$$= \int_0^1 \pi (x^{1/4})^2 dx - \int_0^1 \pi x^2 \, dx$$

$$= \int_0^1 \pi x^{1/2} \, dx - \int_0^1 \pi x^2 \, dx$$

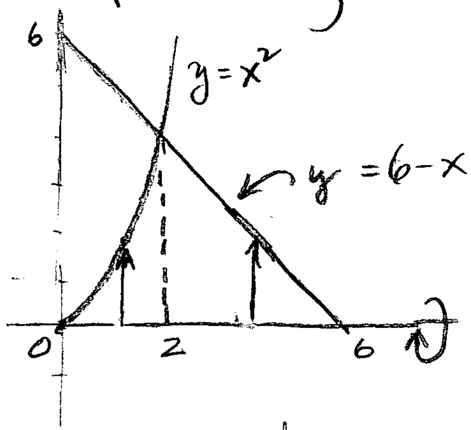
$$= \frac{2\pi x^{3/2}}{3} \Big|_0^1 - \frac{\pi x^3}{3} \Big|_0^1$$

$$= \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

6

Math 131 Day 15

#7



Two pieces!

Intersect: $x^2 = 6-x$

$x^2 + x - 6 = (x-2)(x+3) = 0$

$x = 2, \cancel{-3}$

$$\text{Vol} = \int_0^2 \pi (x^2)^2\,dx + \int_2^6 \pi (6-x)^2\,dx$$

$u = 6-x$

$du = -dx$

$-du = dx$

$x=2 \Rightarrow u=4$

$x=6 \Rightarrow u=0$

$$= \int_0^2 \pi x^4\,dx - \int_4^0 \pi u^2\,du$$

$$= \frac{\pi x^5}{5}\Big|_0^2 + \int_0^4 \pi u^2\,du$$

$$= \frac{\pi \cdot 32}{5} + \frac{\pi \cdot u^3}{3}\Big|_0^4$$

$$= \frac{32\pi}{5} + \frac{64\pi}{3} = \boxed{\frac{416\pi}{15}}$$

#8

The definite integral is a limit of Riemann Sums

$$\int_0^2 1 + \frac{x^2}{2}\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$= \lim_{n\to\infty} \text{Right}(n)$$

$$= \lim_{n\to\infty} \frac{10}{3} + \frac{2}{n} + \frac{2}{n^2} = \frac{10}{3}$$

$\frac{2}{n} \to 0, \quad \frac{2}{n^2} \to 0$