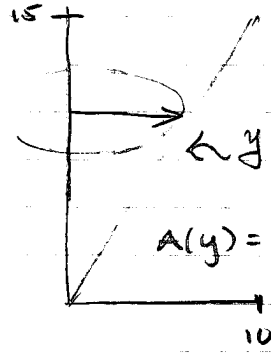
Math 131 Day 19

#1



$W = D\int_0^9 (15-y)A(y)\,dy = 54\int_0^9 (15-y)\pi(\frac{2}{3}y)^2 dy$ (top of oil listed to, bot of oil)

$= 54\pi\int_0^9 (15-y)(\frac{4}{9}y^2)\,dy$

$= 54\pi\int_0^9 \frac{60}{9}y^2 - \frac{4}{9}y^3\,dy = 54\pi\left[\frac{20}{9}y^3 - \frac{1}{9}y^4\right]_0^9$

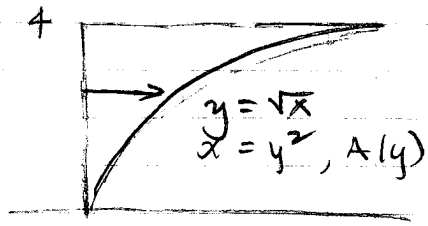
$= 54\pi\left[20(81) - 9^3\right] = \boxed{48114\pi \text{ ft-lbs}}$

b) $W = 54\int_0^{15} (15-y)\pi(\frac{2}{3}y)^2 dy$ ← only change

$= 54\pi\left[\frac{20}{9}y^3 - \frac{1}{9}y^4\right]_0^{15}$

$= 54\pi\left[\frac{20}{9}\cdot 15^3 - \frac{15^4}{9}\right] = \boxed{101,250 \text{ ft-lbs}}$

#2 a)



$W = 65\int_0^4 (4-y)\pi y^4 dy$ ← Full

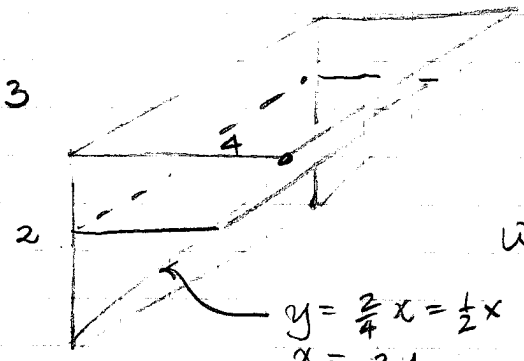
$= 65\pi\int_0^4 4y^4 - y^5\,dy$

$= 65\pi\left[\frac{4}{5}y^5 - \frac{1}{6}y^6\right]\Big|_0^4 = \frac{26624\pi}{3}$ ft-lbs

b) $W = 65\int_2^4 (4-y)\pi(y^4)\,dy = 65\pi\left[\frac{4}{5}y^5 - \frac{1}{6}y^6\right]_2^4$ ← Top 2ft

$= \frac{26624}{3} - \frac{2912}{3} = 7904\pi$ ft-lbs

#3



Rectangle $= A(y) = 2 \cdot y \cdot 8$ (2 ← width, 8 ← length)

$A(y) = 16y$

$W = 62.5\int_1^2 (2-y)(16y)\,dy = 1000\int_1^2 2y - y^2\,dy$ (1 ← Top ft)

$= 1000\left[y^2 - \frac{1}{3}y^3\right]_1^2$

$= 1000\left[(4 - 8/3) - (1 - 1/3)\right] = \frac{2000}{3}$ ft-lbs

#2c) $W = 65\int_0^4 (7-y)\pi y^4\,dy$ ← 3ft above top

#4 $\int x\ln x\,dx = uv - \int v\,du = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2\ln x - \int \frac{x}{2}\,dx$

$= \boxed{\frac{1}{2}x^2\ln x - \frac{x^2}{4} + C}$