Math 131 Day 20

#1) f $\quad \int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2} + c = \frac{1}{2}\ln^2 x + c$

$u = \ln x,\ du = \frac{1}{x}\,dx$

h) $\int (x^2+1)e^x\,dx = (x^2+1)e^x - \int 2xe^x\,dx = (x^2+1) - \left[2xe^x - \int 2e^x\,dx\right]$

$u = x^2+1 \quad dv = e^x dx \qquad u = 2x \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x \qquad du = 2dx \quad v = e^x$

$= (x^2+1)e^x - 2xe^x + 2e^x = (x^2-2x+3)e^x + c$

m) $\int \frac{1}{\sqrt{1-9x^2}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{3}\arcsin u + c = \frac{1}{3}\arcsin(3x) + c$

$u^2 = 9x^2 \quad u = 3x \quad du = 3dx,\ \frac{1}{3}du = dx$

#2 p 520 #12

$\int s e^{-2s}\,ds = -\frac{1}{2}se^{-2s} - \int -\frac{1}{2}e^{-2s}\,dx = -\frac{1}{2}se^{-2s} - \frac{1}{4}e^{-2s} + c$

$u = s \quad dv = e^{-2s}dx$

$du = ds \quad v = -\frac{1}{2}e^{-2s}$

or use substition $w = x^2,\ dw = 2x\,dx,\ \frac{1}{2}dw = x\,dx$

Now do $\frac{1}{2}\int \arctan(w)\,dw$

p 520 #22

$\int x\tan^{-1}(x^2)\,dx = \frac{1}{2}x^2\arctan(x^2) - \int \frac{x^3}{1+x^4}\,dx$

$u = \arctan(x^2) \quad dv = x\,dx$

$du = \frac{2x}{1+x^4}\,dx \quad v = \frac{1}{2}x^2$

$u = 1+x^4$

$du = 4x^3 dx$

$\frac{1}{4}du = x^3 dx$

$= \frac{1}{2}x^2\arctan(x^2) - \int \frac{1}{4}\frac{1}{u}\,du$

$= \frac{1}{2}x^2\arctan(x^2) - \frac{1}{4}\ln|u| + c$

$= \frac{1}{2}x^2\arctan(x^2) - \frac{1}{4}\ln|1+x^4| + c$

p520 #14 $\quad \int \theta\sec^2\theta\,d\theta = \theta\tan\theta - \int \tan\theta\,d\theta$

$u = \theta \quad dv = \sec^2\theta\,d\theta \qquad = \theta\tan\theta - \ln|\sec\theta| + c$

$du = d\theta \quad v = \tan\theta$

p520 #18 $\quad \int \arcsin x\,dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\arcsin x + \int \frac{1}{2}u^{-1/2}\,du$

$u = \arcsin x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{1}{2}du = -x\,dx$

$= x\arcsin x + u^{1/2} + c$

$= x\arcsin x + \sqrt{1-x^2} + c$

p520 #26

$\int x^2 \ln^2 x\, dx = \frac{x^3}{3} \ln^2 x - \int \frac{2x^2}{3} \ln x\, dx$

$u = \ln^2 x$ $\quad dv = x^2 dx$

$du = \frac{2 \ln x}{x} dx$ $\quad V = \frac{x^3}{3}$

$u = \ln x$ $\quad dv = \frac{2x^2}{3} dx$

$du = \frac{1}{x} dx$ $\quad V = 2/9\, x^3$

$= \frac{x^3}{3} \ln^2 x - \left[\frac{2}{9} x^3 \ln x - \int \frac{2}{9} x^2 dx \right]$

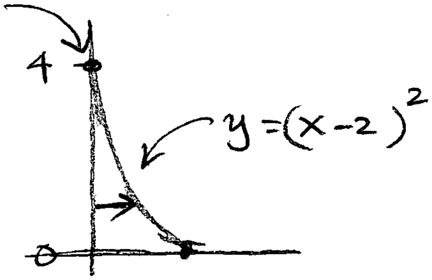
$= \frac{x^3}{3} \ln^2 x - \frac{2}{9} x^3 \ln x + \frac{2}{27} x^3 + C$

#3 Work $\quad y = (x-2)^2 \Rightarrow \sqrt{y} = x - 2 \Rightarrow x = \sqrt{y} + 2$

$A(y) = \pi x^2 = \pi(\sqrt{y} + 2)$

$= \pi(y + 4\sqrt{4} + 4)$

Top of tank @ $x = 0$ ... $y = (0-2)^2 = 4$



Top layer of liquid

a) $W = \int_0^4 \pi(y + 4\sqrt{y} + 4)[4 - y]\,dy$

bot of liquid $\quad$ move to

2 feet above top

b) $W = \int_0^4 \pi(y + 4\sqrt{y} + 4)[6 - y]\,dy$

top of oil $\quad$ move to top of tank

c) $W = \int_0^1 \pi(y + 4\sqrt{y} + 4)[4 - y]\,dy$