Math 131 Day 22

#1

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

$\cos^2(3\theta) = \frac{1}{2} + \frac{1}{2}\cos(6\theta)$

$\sin^2(4\theta) = 1/2 - 1/2\cos(6\theta)$

$\cos^2(-2\theta) = 1/2 + 1/2\cos(-4\theta)$

#2a) $\int \sin^2(4t)\,dt = \int \frac{1}{2} - \frac{1}{2}\cos(8t)\,dt = \frac{1}{2}t - \frac{1}{16}\sin(8t) + c$

b) $\int \cos^5(3x)\,dx = \frac{1}{3}\int \cos^5 u\,du = \frac{1}{3}\left[\frac{1}{5}\cos^4 u \sin u + \frac{4}{5}\int \cos^3 u\,du\right]$

$u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$

$= \frac{1}{3}\left[\frac{1}{5}\cos^4 u \sin u + \frac{4}{5}\left[\frac{1}{3}\cos^2 u \sin u + \frac{2}{3}\int \cos u\,du\right]\right]$

$= \frac{1}{3}\left[\frac{1}{5}\cos^4 u \sin u + \frac{4}{5}\left[1/3\cos^2 u \sin u + \frac{2}{3}\sin u\right]\right] + C$

$= \frac{1}{15}\cos^4 3x \sin 3x + \frac{4}{45}\cos^2 3x \sin 3x + \frac{8}{15}\sin(3x) + C$

c) $\int \sin^2(4x)\cos^3(4x)\,dx = \int \sin^2(4x)\cos^2(4x)\cos(4x)\,dx$

$= \int \sin^2(4x)\left(1 - \sin^2(4x)\right)\cdot\cos(4x)\,dx$

$u = \sin 4x$
$du = 4\cos 4x\,dx$
$1/4\,du = \cos 4x\,dx$

$= \frac{1}{4}\int u^2(1-u^2)\,du = 1/4\int u^2 - u^4\,du$

$= 1/4\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + c = \frac{\sin^3(4x)}{12} - \frac{\sin^5(4x)}{20} + c$

d) $\int \sin^3(2x)[\cos(2x)]^{-4}\,dx = \int \sin^2(2x)[\cos(2x)]^{4}\sin(4x)\,dx$

$u = \cos 2x$
$-1/2\,du = \sin(2x)\,dx$

$= \int (1-\cos^2 2x)[\cos(2x)]^{-4}\sin(4x)\,dx$

$= -\frac{1}{2}\int (1-u^2)u^{-4}\,du = -\frac{1}{2}\int u^{-4} - u^{-2}\,du$

$= -1/2\left[-\frac{u^{-3}}{3} + u^{-1}\right] + c$

$= \frac{[\cos(2x)]^{-3}}{6} - \frac{[\cos(2x)]^{-1}}{2} + c = \frac{(\cos 2x)^{-3}}{6} - \frac{1}{2\cos(2x)} + c$

e) $\int \sin^2(2x)\cos^2(2x)\,dx = \int (1/2 - 1/2\cos(4x))(1/2 + 1/2\cos(4x))\,dx$ ← The original problem

$= \int 1/4 - 1/4(\cos^2(4x))\,dx = \int 1/4 - 1/4\left[1/2 + \frac{1}{2}\cos^2(8x)\right]dx$

$= \int 1/4 - 1/8 - 1/8\cos(8x)\,dx = \frac{x}{8} - \frac{1}{64}\sin(8x) + C$